

# The Reforms of Shang Yang

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## Abstract

This paper studies the Reforms of Shang Yang in a principal-agent model. Shang Yang tried to rationalize government administration and to organize the economy more efficiently. Those reforms laid the foundation of Qin's unification of China. In this paper, the ruler chooses incentives for soldiers, the degree of the adoption of the county form, and the level of military spending to handle internal rebellions by nobles and external threats. The existence of institutional complementarities between the adoption of the county form and the use of strong incentives for soldiers is established. An increase in the level of external threats or the degree of risk aversion of soldiers induces the ruler to adopt stronger incentives for soldiers and to choose a higher degree of military spending and a higher degree of adopting the county form while the ruler's personal consumption does not change. An increase in the power of clans leads the ruler to choose a lower incentive for soldiers and a lower level of adopting the county form.

**Keywords:** Institutional reform, Chinese history, political economy, national strategy of governance, principal-agent model

**JEL Classification Numbers:** N45, H10, D80, P40

## 1. Introduction

Many countries in different periods engaged in reforms which were associated with significant reallocation of costs and benefits among players. The Reforms of Shang Yang during the Warring States period (475 BC-221 BC) was a significant event in China's history. At that time, states competed fiercely to survive. Through measures reducing transaction costs and rationalizing government organization, Shang Yang made the economy of the state of Qin more efficient. The Reforms were successful and laid the foundation of Qin's unification of China in 221 BC.<sup>1</sup> Even though that the Qin dynasty did not last long, institutions such as the commandary-county government organizational form (simplified as the county form in the rest of this paper) in the following dynasties were based on Qin institutions. Since the Reforms of Shang Yang affected the evolution of Chinese institutions in a fundamental way, Shang Yang is regarded as the chief architect of the Chinese state (Li, 1977, p. xiv). Various important issues highlighted in the Reforms are still relevant for today's institutional reforms. First, one issue is the existence of

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<sup>1</sup> To explain the state of Qin's unification of China, Zhou (2021) has presented a dynamic model in which culture affects the choice of institutions while institutions affect the evolution of culture.

institutional complementarity among different reform measures. As happened in other states in the Warring States period discussed in Section 2, without mechanisms such as using the county form to establish the centralization of power at the national government, introducing strong incentives to soldiers in handling external threats would lead to accumulation of power of generals and thus would be self-defeating for rulers. With the existence of institutional complementarities among reform measures, in general Shang Yang's reforms were coherent. Second, culture affects institutional reforms. Qin's deviation from Confucianism made it possible for Qin to engage in comprehensive reforms.

While the Reforms of Shang Yang has been discussed extensively, the literature on this issue is descriptive. To our limited knowledge, there is no mathematical model studying the Reforms of Shang Yang. A mathematical model will be valuable for us to organize our thinking on this important issue. From measures such as rationalizing government organization and making efficient use of resources in the Reforms of Shang Yang, we can see a strong desire for efficiency in Shang Yang's design of institutions. Economics focuses on efficiency. Thus, economics is an appropriate tool to study the Reforms of Shang Yang. This paper studies the Reforms of Shang Yang by embedding a principal-agent model into the maximization problem of a ruler (the principal). In this model, the ruler faces internal rebellions and external threats and chooses the level of incentives for soldiers (the agent), the level of military spending, and the degree of adoption of the county form to maximize his expected payoff. The ruler faces participation constraint of an agent and the resource constraint.

We establish the existence of institutional complementarity between the provision of incentives to soldiers and the adoption of the county form. Historically, the adoption of the county system increased the central government's control on local governments and the standardization of measurement units diminished variance of performance measurement. Thus, adopting the county form reduces the possibility of internal rebellions when soldiers are provided with strong incentives because successful fighters would not be able to accumulate their family influence under the county form.

We show that a higher level of external threats or a higher degree of risk aversion of soldiers induces the ruler to choose a higher level of incentives for soldiers, a higher level of military spending, and a higher level of adopting the county form while the ruler's personal consumption does not change. When clans are stronger, we show that the ruler chooses a lower

level of personal consumption, a lower level of incentives for soldiers, a lower military spending, and a lower level of adopting the county form. Interestingly, the influence of some factors such as the level of population size on the ruler's provision of incentives to soldiers turns out to be unexpected even in this model: a higher population induces the ruler to choose a lower rather than a higher level of incentives for soldiers.

This paper is related to the literature on design of reform packages. For this line of literature, McKinnon (1993) has studied the order of financial reform and trade reform in economic development. Motivated by transition experience in East Europe, Dewatripont and Roland (1995) have compared gradualist reform packages with big-bang strategies. While China adopts a gradual approach in transition from a planned economy to a market economy (Qian, 2017), Russia takes a more dramatic approach in transition through implementation of large-scale privatization in the 1990s. The specifications of the above studies are significantly different from that in this model. The reforms in the Warring States periods are consistent with observations in transition countries in the sense that reforms would be more effective if formal institutions are consistent with culture and social norms.

This paper is related to the literature on principal-agent models. For this line of literature, Holmstrom (1979) has explored incentive provision in a model in which the principal maximizes expected benefit subject to the participation constraint and the incentive compatibility constraint of an agent. Holmstrom and Milgrom (1987) have addressed the optimality of linear incentive contracts. In terms of application on economic history, Ma and Rubin (2019) have studied a model in which the ruler as the principal hires an agent to collect taxes and the agent chooses the effort level. Interestingly, to ensure that no predation becomes credible, the ruler in their model chooses not to invest in administrative capacity. While the adoption of the principal-agent approach is similar, one significant difference between this model and Ma and Rubin (2019) is that internal rebellions and external threats are not addressed in their model.

This paper is related to the literature on institutional complementarities. For this line of literature, Milgrom and Roberts (1990) have studied the existence of institutional complementarities in modern production. Aoki (2001) has demonstrated the existence of institutional complementarity in the Japanese economic system. Zhou (2018) has studied complementarities among the adoption of the county form, the usage of the imperial examination system in selecting officials, and the division of power among government officials in imperial

China. One significant difference between this model and Zhou (2018) is that incentives of soldiers are not addressed in Zhou (2018).

The plan of the paper is as follows. Section 2 briefly illustrates historical evidence of the Reforms of Shang Yang to motivate key variables and parameters of the model. Section 3 specifies the model and derives the set of optimal conditions for the ruler's maximization. Section 4 conducts comparative statics to explore how various key parameters such as the level of external threats and population size affect the ruler's choices of incentives to soldiers, military spending, and the level of adoption of the county form. Section 5 concludes.

## 2. The Reforms of Shang Yang

Sima (1988) contains historical records of the Reforms of Shang Yang. Li (1977) provides illustration and translation of some related documents. The Book of Lord Shang<sup>2</sup> contains ideas of Shang Yang. However, it is generally believed that it was not written by Shang Yang because this book refers to events happened much later after the death of Shang Yang.

The Reforms of Shang Yang happened in the Warring States period. At that time, the use of iron increased agricultural productivity and led to population growth. With higher population, increased demand for land led to wars among states. With the power of the Zhou ruling house declined, states fight among themselves to survive wars of annexation.

Compared with other major states in that period, the state of Qin was relatively backward (Lin, 1981). Duke Xiao of Qin inherited his position in 362 BC. Faced with fierce competition from other states, he decided to attract talents so that the state could recover her former glories. Shang Yang, a native of Wei (卫),<sup>3</sup> was an important figure of the Legalist school of thoughts in ancient China. Initially he worked as a staff for prime minister Gongshu Zuo in the state of Wei (魏), however the ruler in Wei (魏) did not follow Gongshu Zuo's suggestion to let Shang Yang to replace Gongshu's position. Knowing that Duke Xiao in the state of Qin was recruiting, Shang Yang decided to go to Qin to try his fortune. With reforms conducted by Li Qui (李悝), Wei (魏) was the strongest state at that time. Shang Yang's experience at Wei (魏) should be helpful for his understanding of state-of-art practice of policies.

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<sup>2</sup> *The Book of Lord Shang (商君书)*, 2011, translated by Lei Shi, Beijing, China: China Book Company.

<sup>3</sup> Wei could be either 卫 or 魏, both were names of states in the Warring States period. Chinese character will be added if there is potential confusion of a name or a location.

## 2.1. The reform process

With the intermediation of an eunuch in the state of Qin, Shang Yang got the opportunity of several rounds of interviews with Duke Xiao during which Shang Yang offered a menu of grand strategies to Duke Xiao. Duke Xiao was not interested in the imperial way (帝道) and the benevolent rule (王道), but he hired Shang Yang to implement the way of the hegemon (霸道) to get immediate results for Qin so that Qin could compete with other states (Sima, 1988).

In 359 BC, Duke Xiao held a meeting of government officials to discuss whether to reform. Those who opposed reform pointed out risks associated with reform. The young Duke Xiao sided with Shang Yang that institutions should be consistent with contemporary demands. Started at 359 BC, Shang Yang conducted two rounds of reforms in the state of Qin. For the first round of reform, Shang Yang reformed household registration system and a household would be responsible for behavior of neighbors. One important change was that nobles need military achievements to be granted with important government positions. Military achievements would be rewarded through a system of twenty scales. Shang Yang also tried to change Qin customs. For example, individuals were discouraged from fighting for personal reasons and they should fight for the state. In addition, commercial activities were heavily taxed (Sima, 1988). While controversial, this measure was justified by arguing that merchants did not produce agricultural goods and thus unproductive to society.

For the second round of reform, the state of Qin moved its capital from Yueyang (栎阳, located at Lintong District of Xi'an City today) to Xianyang. Since not all nobles moved to the new capital, the influence of nobles in the new capital was smaller. First, small towns and villages were combined into counties and the commandery-county system was implemented across the state (Zhang, 2004). Those measures rationalized government organization. Second, Shang Yang unified measures and the government distributed standard measures across the whole state (Sima, 1988). This standardization would be helpful in reducing variance in evaluating performance of government officials. Third, the state of Qin adopted Legalism as the national strategy of governance (Zhao, 2015).<sup>4</sup> Shang Yang believes that laws should be simple and easy to understand. Government officials should explain laws to citizens clearly. Other schools of thoughts

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<sup>4</sup> Zhou (2011) has illustrated pros and cons of Confucianism and the Legalism as a national strategy of governance. He shows that a combination of Confucianism and the Legalism could perform better than either one is adopted.

such as Confucianism were discouraged. Fourth, the Qin government also took measures to increase land through reclaiming land and to increase population through increasing birth rates and attracting immigrants from other states. Different from the previous well-field system, land could be privately owned and sold.

While the first round of reform targeted at immediate issues like military performance and agricultural production, the second round of reform tried to synthesis the Qin system. Overall, reform measures increased efficiency, and resources were channeled effectively to fight wars.

Since handling interstate wars was the most demanding issue at that time, it was not strange that Shang Yang was also a successful military leader. He led the Qin army defeated Wei (魏) (Sima, 1988). Bamboo records from Hubei show that Qin had detailed institutions in military management,<sup>5</sup> which was likely to be a result of the Reforms. One advantage for Shang Yang as an outsider of Qin politics was that he was not intertwined with powerful clans in Qin. This made it easier for him to carry out the reforms. Shang Yang's reforms harmed interests of nobles, and some opposed his reform measures. However Shang Yang was able to remain in power with the support of Duke Xiao backed with strong state power. After the death of Duke Xiao in 338 BC, the new ruler (King Hui) might have worried that Shang Yang was too powerful, and Shang Yang was sentenced to death. However, King Hui kept the reform measures introduced by Shang Yang.

While there was redistribution during the Reforms, the size of the pie also increased during the Reforms. The Reforms increased agricultural production (the most important sector at that time) in Qin significantly. With the foundation laid by the Reforms, the state of Qin succeeded in unifying China in 221 BC. The impact of the Reforms can be seen from the following statistics. After the Reforms of Shang Yang, Qin won 58 wars very successfully out of 65 major wars before unifying China (Yu, 2015, p. 204).

## **2.2. Comparisons with reforms in other states**

Qin was not the only state that used strong incentives in rewarding soldiers. In the Warring States period, all the seven major states engaged in some kinds of reforms. Otherwise, they could not have survived. We can improve our understanding of the Reforms of Shang Yang by

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<sup>5</sup> Source: Shuihudi Qin bamboo texts 《睡虎地秦墓竹简》, 1974. 睡虎地秦墓竹简整理小组编. Beijing, China: Wenwu Publishing House.

comparing reforms in the state of Qin with those in other states. Why was Qin more successful in reforming?

First, during the Spring and Autumn period, Jin (晋) rewarded ministers with military achievements. This produced unexpected results under Jin's political system. Huan Shu of Quwo (曲沃桓叔) (802 BC-731BC) was not the oldest son of his father, Marquis Mu of Jin (晋穆侯). Thus, Huan Shu could not inherit his father's position. However, Huan Shu's father liked him and Huan Shu was awarded the land in Quwo, a strategic location. This planted the seed of an internal rebellion in Jin. Later, Huan Shu's grandson (Duke Wu of Jin) usurped Jin (Sima, 1988). To prevent this kind of usurpation of power by relatives from happening again, Jin tried to reduce the power of princes through measures such as sending princes abroad. In Jin, with the influence of princes reduced, strong incentives to military performance without the county system led to the rise of powerful generals unrelated to the royal family: After each successful war, generals were awarded more land. This eventually led to the division of Jin into three states: Wei (魏), Zhao, and Han (韩). The key difference between Jin and Qin was that land awarded was inheritable and the military was controlled by ministers in Jin. In Qin, nobles had economic benefits through revenue collected but they did not control the military. For example, Shang Yang could not control the military in the land awarded to him.

Second, another comparison is between Qin and Wei (魏) in terms of military institutions which demonstrates the importance of sustainability of military strategy. Under Marquis Wen, the state of Wei (魏) had institutional reforms. One of the reform measures was the introduction of "military soldiers" (武卒). Wei's military soldiers were trained for several years and heavily armed. With her superb soldiers, Wei (魏) was the strongest state at the beginning of the Warring States period. However, this system was expensive for Wei (魏) (Xunzi, 2011, p. 231). Also, since heavy training was needed, it was difficult to replace dead soldiers. The small number of military soldiers was not suitable for the transition to large-scale wars. Qin's reward of soldiers was consistent with large-scale wars.<sup>6</sup> At that period, scales of war became large. For example, during the War of Changping, 400,000 defeated soldiers of Zhao were killed (Sima, 1988).

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<sup>6</sup> This may be like the replacement of city states with nation states when large-scale wars using peasant soldiers replacing knights happened in Europe.

Third, in the state of Chu, Wu Qi conducted reform under King Dao. Wu Qi's reform focused on military, and economic and social reforms were limited. After the death of King Dao, Wu Qi was killed by nobles (Sima, 1988). The new king reversed some reform measures to please nobles. In the state of Chu, politics was controlled by three noble families and high-rank government positions were monopolized by those families. While the royal family was not usurped as happened in Jin and Qi (齐), comprehensive reforms could not be started and sustained as shown in the death of Wu Qi.

### **2.3. Interpretation of the Reforms of Shang Yang**

Like the scientific management movement in the twentieth century, Shang Yang tried to conduct businesses efficiently. With the establishment of formal law and the unification of tax, military, and government administration, the reform process in the state of Qin was also a transition process from relation-based governance to rule-based governance (Zhou, 2021).<sup>7</sup> The Reforms of Shang Yang demonstrated the power of institutions. Complementarities among standardization of instruments, the adoption of the county form, and the provision of strong incentives to soldiers might have existed. Under the county form, government officials are appointed by the central government rather than inherited. The county form reduces the power accumulation of ministers and make strong incentives to soldiers unlikely to lead to internal rebellions. Standardization of instruments reduces measurement errors and decreases the distortion from the use of strong incentives.<sup>8</sup>

In China's history, there are quite different views of Shang Yang. On the negative side, Shang Yang is claimed to be mean (Sima, 1988), consistent with the cold-blood image of Legalist school. His reform measures contributed to the concentration of power on rulers in Imperial China. On the positive side, Shang Yang helped the establishment of a bureaucracy and rule-based institutions. Qin institutions influenced future dynasties and Shang Yang is viewed as the chief architect of the Chinese state (Li, 1977, p. xiv).

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<sup>7</sup> The Warring States period was a process of national integration, and transformations from relation-based governance to rule-based governance happened in this period. Zhou (2024) demonstrates the mutual dependence between national integration and the switch from relation-based to rule-based institutions in a formal model.

<sup>8</sup> There were fixed costs of standardization. With the promotion of the county form, fixed costs of standardization could be spread to a larger region and average fixed cost decreases.



Overall, while Shang Yang's comprehensive reforms were based on reforms happened in other states, Shang Yang took into consideration of Qin's specific situations such as relatively high land-labor ratio in designing his reforms. The success highlighted the importance of institutional complementarities and the impact of culture on institutional choice. Qin's reform was consistent with the historical trend of replacing feudalism<sup>9</sup> with the county system. Qin's success in reforming had historical roots. In the Shang dynasty, Qin was an ally of Shang. Confucianism had limited influence in Qin, and this reduced power of nobles who would oppose reforms (Lin, 1981; Zhou, 2021). Under Confucianism, nobles inherited land and power. Different from Confucianism, an important feature of Qin culture is that the patriarchal clan system was not strictly enforced in Qin (Lin, 1981, p. 80, p. 85). Qin's culture reduced power of nobles and made comprehensive reforms possible.

### 3. The model

There is a ruler who is risk neutral. The exogenous population size is  $L$ . An individual can be either a soldier or a peasant, but not both. Soldiers participate in wars and peasants produce the agricultural good. A fixed percentage  $b$  of the population will be soldiers. This can be motivated by arguing that whether an individual is a soldier is determined by age, rather than by individual choice. Soldiers are risk averse. The amount of exogenously given land is  $T$ .

The timing of the game is as follows. First, the ruler chooses incentive for soldiers, anticipating the behavior of a representative soldier.<sup>10</sup> The ruler's payoff is affected by internal rebellions by nobles and external threats (Zhou, 2012).<sup>11</sup> Second, given the incentive schemes, a representative soldier chooses effort level. Third, the ruler's expected payoff is realized.

#### 3.1. Incentive of soldiers

The effort of a soldier is not observable to the ruler but output is observable. The incentive contract the ruler offers to a soldier is assumed to be linear because a linear incentive contract is less likely to be manipulated, as shown by Holmstrom and Milgrom (1987). They demonstrate that

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<sup>9</sup> Scholars have different understanding of the meaning of feudalism in China. In this paper, feudalism in China refers to the period under Zhou ruling house.

<sup>10</sup> Shang Yang represented the ruler when reforming institutions.

<sup>11</sup> Zhou (2012) studies a ruler's choice of government organizational form to handle internal rebellions and external threats.

linear incentive contracts could be optimal if the agent has a constant degree of risk aversion and uncertainty is normally distributed, which are assumed in this model. If the level of output of a soldier is  $z$ , the reward to a soldier is specified as

$$s(z) = \rho z + \eta. \quad (1)$$

Here,  $\rho$  measures the strength of incentives. A higher value of  $\rho$  means that the ruler provides a stronger incentive for soldiers to fight harder. The level of reservation utility of a soldier is  $R$ , which is exogenously given. Through the choice of  $\eta$ , the contract is designed so that the participation constraint of a soldier is satisfied.<sup>12</sup>

For  $r$  denoting a positive constant, a soldier's utility function is  $U(q) = -e^{-rq}$ , where  $q$  is a soldier's payoff. Thus, the degree of risk aversion for a soldier is  $r$ . When the level of effort of a soldier is  $\mu$ , what the ruler observes is  $\mu + \varepsilon$ , where  $\varepsilon$  is a random variable with a normal distribution. The mean of  $\varepsilon$  is zero and the variance of the normal distribution is  $\sigma^2$ . The cost of effort for a soldier is  $\frac{k}{2}\mu^2$ , where  $k$  is a positive constant. Because uncertainty has a normal distribution, a soldier's expected payoff is

$$\rho\mu + \eta - \frac{k}{2}\mu^2 - \frac{r}{2}\rho^2\sigma^2. \quad (2)$$

A soldier chooses effort optimally to maximize expected payoff (2). A soldier's optimal choice of effort yields

$$k\mu = \rho. \quad (3)$$

Since  $k$  is a positive constant, a higher value of  $\mu$  means that the ruler provides a stronger incentive to soldiers. For a soldier's participation constraint to be satisfied, the ruler chooses  $\eta$  such that  $\eta = \frac{r}{2}k^2\mu^2\sigma^2 - \frac{k}{2}\mu^2 + R$ . Thus, from (1), for the ruler to induce a soldier to exert effort level  $\mu$ , the cost for the ruler is  $\frac{r}{2}k^2\mu^2\sigma^2 + \frac{k}{2}\mu^2 + R$ .

### 3.2. The ruler's maximization problem

When total effort of  $bL$  soldiers is  $\mu bL$ , the level of military spending is  $m$ , and the level of external threats is  $\Omega$ , the ruler handles external threats successfully with probability  $\frac{\mu bL m}{\mu bL m + \Omega}$ .<sup>13</sup>

<sup>12</sup> Even though the government has coercive power, the participation constraint is assumed to be satisfied because citizens can move to other states.

<sup>13</sup> This specification of the probability of winning follows the rent-seeking literature.

The value to the ruler of staying in power if external threats are handled successfully is his personal consumption. If the ruler does not handle external threats successfully, his payoff is zero.

Soldiers will not be able to engage in agricultural production. Thus, the number of individuals producing the agricultural good is  $(1 - b)L$ . For  $\alpha$  and  $\beta$  denoting positive constants, total agricultural output is  $[(1 - b)L]^\alpha T^\beta$ . Since the ruler may not control all land under feudalism, not all agricultural output is available to the ruler. The ruler controls output of counties only. For  $x$  denoting the degree of adoption of the county form, the amount of agricultural output available for the ruler is  $x[(1 - b)]L^\alpha T^\beta$ .

For  $a$  denoting a positive constant, the probability of rebellion by nobles is  $1 - e^{-ax}$ . A higher value of  $a$  means that nobles are more likely to rebel. That is, a higher level of adoption of the county form increases the percentage of agricultural output available to the ruler. However, nobles are more likely to rebel if the ruler tries to adopt a higher degree of county form. This assumption can be motivated as follows. Under feudalism, positions of Great Officers are hereditary, and the ruler could not control resources of Great Officers. Under the county form, the ruler appoints county officials and has more control of resources. The adoption of the county form will harm the interests of nobles and thus they are more likely to rebel.

The ruler's consumption is  $C$ . The ruler pays each soldier  $\frac{r}{2}k^2\mu^2\sigma^2 + \frac{k}{2}\mu^2 + R$  and total payment to  $bL$  soldiers is  $\frac{r}{2}k^2\mu^2\sigma^2 bL + \frac{k}{2}\mu^2 bL + RbL$ . The budget constraint faced by the ruler is that total rewarding to soldiers, military spending, and consumption of the ruler should not be higher than (equals) total revenue:

$$\frac{r}{2}k^2\mu^2\sigma^2 bL + \frac{k}{2}\mu^2 bL + RbL + m + C = x[(1 - b)L]^\alpha T^\beta. \quad (4)$$

The ruler handles external threats successfully with probability  $\frac{bL\mu m}{bL\mu m + \Omega}$  and he handles internal rebellions successfully with probability  $e^{-ax}$ . His expected payoff is  $\frac{bL\mu m}{bL\mu m + \Omega} e^{-ax} \ln C$ . By using the budget constraint (4), the ruler's expected payoff can be written as

$$\Pi \equiv \frac{bL\mu m}{bL\mu m + \Omega} e^{-a} \ln \left( x[(1 - b)]L^\alpha T^\beta - \frac{r}{2}k^2\mu^2\sigma^2 bL - \frac{k}{2}\mu^2 bL - RbL - m \right). \quad (5)$$

The ruler chooses the level of incentives for soldiers, the level of military spending, and the degree of the adoption of the county form to maximize expected payoff (5).<sup>14</sup> The ruler could

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<sup>14</sup> To address unification and division in imperial China, Zhou (2023) has analyzed a ruler's choice of economic and political institutions such as tax rates to handle internal rebellions by peasants and external threats. One significant

have corner solutions when the maximum value of adopting the county form is chosen. That is, all local administrations are organized as counties. Assuming interior solutions, the first order conditions with respect to  $\mu$ ,  $m$ , and  $x$  are respectively<sup>15</sup>

$$\begin{aligned} \frac{\Omega}{bL\mu m + \Omega} \ln \left( x[(1-b)L]^\alpha T^\beta - \frac{r}{2}k^2\mu^2\sigma^2bL - \frac{k}{2}\mu^2bL - RbL - m \right) \\ - \frac{\mu bL(k\mu + rk^2\sigma^2\mu)}{x[(1-b)L]^\alpha T^\beta - \frac{r}{2}k^2\mu^2\sigma^2bL - \frac{k}{2}\mu^2bL - RbL - m} = 0, \end{aligned} \quad (6a)$$

$$\begin{aligned} \frac{\Omega}{bL\mu} \ln \left( x[(1-b)L]^\alpha T^\beta - \frac{r}{2}k^2\mu^2\sigma^2bL - \frac{k}{2}\mu^2bL - RbL - m \right) \\ - \frac{m}{x[(1-b)L]^\alpha T^\beta - \frac{r}{2}k^2\mu^2\sigma^2bL - \frac{k}{2}\mu^2bL - RbL - m} = 0, \end{aligned} \quad (6b)$$

$$\begin{aligned} \frac{[(1-b)L]^\alpha T^\beta}{x[(1-b)L]^\alpha T^\beta - \frac{r}{2}k^2\mu^2\sigma^2bL - \frac{k}{2}\mu^2bL - RbL - m} \\ - a \ln \left( x[(1-b)L]^\alpha T^\beta - \frac{r}{2}k^2\mu^2\sigma^2bL - \frac{k}{2}\mu^2bL - RbL - m \right) = 0. \end{aligned} \quad (6c)$$

Equations (6a)-(6c) form a system of three equations defining three variables  $\mu$ ,  $m$ , and  $x$  as functions of exogenous parameters. For equation (6a), the first term is the marginal benefit of providing incentives to soldiers, and the second term is the marginal cost. For equation (6b), the first term is the marginal benefit of military spending, and the second term is the marginal cost. For equation (6c), the first term is the marginal benefit of adopting the county form, and the second term is the marginal cost.

Complementarity among choice variables exists when the second-order cross derivatives of the objective function are nonnegative (Milgrom and Roberts, 1990). Using this terminology, the following proposition demonstrates the existence of complementarities among the major reform measures introduced by Shang Yang. The adoption of the county form can be captured by an increase in  $x$  and the level of incentives to soldiers can be captured by an increase in  $\mu$ .

**Proposition 1:** There are complementarities among the usage of incentives and the adoption of the county form.

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difference between this paper and Zhou (2023) is that incentive provision to risk-averse agents is not addressed in Zhou (2023).

<sup>15</sup> The corresponding second-order conditions for maximization are satisfied.

Proof: Partial differentiating equation (6a) with respect to  $x$ ,  $\frac{\partial^2 \Pi}{\partial \mu \partial x} > 0$ . Alternatively, partial differentiating equation (6c) with respect to  $\mu$ ,  $\frac{\partial^2 \Pi}{\partial x \partial \mu} > 0$ . ■

From Proposition 1, marginal benefit of providing incentives to soldiers increases with the adoption of the county form. Alternatively, marginal benefit of adopting the county form increases with the level of incentives for soldiers. Proposition 1 provides a rationalization of some of the reform measures introduced by Shang Yang. More reform measures may not necessarily increase the chance of success. However, if they are complementary, it will be beneficial to introduce them together. Proposition 1 can be used to interpret different results between Qin and Jin discussed in Section 2.

#### 4. Comparative statics

To conduct comparative statics, we need to simplify the set of first order conditions. From equations (6a)-(6c), we get<sup>16</sup>

$$m - bL\mu(k\mu + rk^2\sigma^2\mu) = 0, \quad (7a)$$

$$\frac{\Omega}{bL\mu m + \Omega} \ln \left( x[(1-b)L]^{\alpha T\beta} - \frac{r}{2}k^2\mu^2\sigma^2bL - \frac{k}{2}\mu^2bL - RbL - m \right) - \frac{m}{x[(1-b)L]^{\alpha T\beta} - \frac{r}{2}k^2\mu^2\sigma^2bL - \frac{k}{2}\mu^2bL - RbL - m} = 0, \quad (7b)$$

$$[(1-b)L]^{\alpha T\beta} - \frac{am(bL\mu m + \Omega)}{\Omega} = 0. \quad (7c)$$

From equations (4), (7b), and (7c), we can derive the following equation defining the ruler's consumption implicitly:

$$\Lambda \equiv c \ln c - \frac{[(1-b)L]^{\alpha T\beta}}{a} = 0. \quad (8)$$

From equations (7a)-(7c), we get the following system of three equations defining three variables  $\mu$ ,  $m$ , and  $x$  as functions of exogenous parameters:<sup>17</sup>

$$\Gamma_1 \equiv m - bL\mu(k\mu + rk^2\sigma^2\mu) = 0, \quad (9a)$$

<sup>16</sup> Equation (7a) is derived from combining (6a) and (6b). Equation (7c) is derived from combining (6b) and (6c).

<sup>17</sup> Equation (9b) is derived from (7b) by using (6a) to eliminate  $\frac{r}{2}k^2\mu^2\sigma^2bL + \frac{k}{2}\mu^2bL$  and using (7c) to eliminate  $\frac{\Omega}{bL\mu m + \Omega}$ . Equation (9c) is derived from (7c) by using (6a) to eliminate  $\mu$ .

$$\Gamma_2 \equiv \frac{aln\left(x[(1-b)L]^{\alpha}T^{\beta}-RbL-\frac{3}{2}m\right)}{(1-b)L^{\alpha}T^{\beta}} - \frac{1}{x[(1-b)L]^{\alpha}T^{\beta}-RbL-\frac{3}{2}m} = 0, \quad (9b)$$

$$\Gamma_3 \equiv [(1-b)L]^{\alpha}T^{\beta} - am - \frac{am^{3/2}}{\Omega} \sqrt{\frac{bL}{k(1+r\sigma^2)}} = 0. \quad (9c)$$

Partial differentiation of equations (9a)-(9b) with respect to  $\mu, m, x, \Omega, T, L, a, \sigma^2, b, R$ , and  $r$  yields

$$\begin{pmatrix} \frac{\partial \Gamma_1}{\partial \mu} & \frac{\partial \Gamma_1}{\partial m} & 0 \\ 0 & \frac{\partial \Gamma_2}{\partial m} & \frac{\partial \Gamma_2}{\partial x} \\ 0 & \frac{\partial \Gamma_3}{\partial m} & 0 \end{pmatrix} \begin{pmatrix} d\mu \\ dm \\ dx \end{pmatrix} = - \begin{pmatrix} 0 \\ 0 \\ \frac{\partial \Gamma_3}{\partial \Omega} \end{pmatrix} d\Omega - \begin{pmatrix} 0 \\ \frac{\partial \Gamma_2}{\partial a} \\ \frac{\partial \Gamma_3}{\partial a} \end{pmatrix} da - \begin{pmatrix} 0 \\ \frac{\partial \Gamma_2}{\partial T} \\ \frac{\partial \Gamma_3}{\partial T} \end{pmatrix} dT \\ - \begin{pmatrix} \frac{\partial \Gamma_1}{\partial L} \\ \frac{\partial \Gamma_2}{\partial L} \\ \frac{\partial \Gamma_3}{\partial L} \end{pmatrix} dL - \begin{pmatrix} \frac{\partial \Gamma_1}{\partial \sigma^2} \\ 0 \\ \frac{\partial \Gamma_3}{\partial \sigma^2} \end{pmatrix} d\sigma^2 - \begin{pmatrix} \frac{\partial \Gamma_1}{\partial b} \\ \frac{\partial \Gamma_2}{\partial b} \\ \frac{\partial \Gamma_3}{\partial b} \end{pmatrix} db - \begin{pmatrix} 0 \\ \frac{\partial \Gamma_2}{\partial R} \\ 0 \end{pmatrix} dR - \begin{pmatrix} \frac{\partial \Gamma_1}{\partial r} \\ 0 \\ \frac{\partial \Gamma_3}{\partial r} \end{pmatrix} dr. \quad (10)$$

Let  $\Delta$  denote the determinant of the coefficient matrix of endogenous variables of (10):

$$\Delta \equiv -\frac{\partial \Gamma_1}{\partial \mu} \frac{\partial \Gamma_2}{\partial x} \frac{\partial \Gamma_3}{\partial m}.$$

Since  $\frac{\partial \Gamma_1}{\partial \mu} < 0$ ,  $\frac{\partial \Gamma_2}{\partial x} > 0$ , and  $\frac{\partial \Gamma_3}{\partial m} < 0$ , it is clear that  $\Delta < 0$ . With  $\Delta$  nonsingular, a unique equilibrium exists. With the existence of a unique equilibrium established, we now proceed to comparative statics.

In China's history such as in the Song dynasty, an increase in the level of external threats has been emphasized as a reason for a state to reform. The following proposition studies the impact of a change in the level of external threats on the ruler's choices.

**Proposition 2:** An increase in the level of external threats leads the ruler to choose a higher level of incentives for soldiers, a higher military spending, and a higher level of adopting the county form. The ruler's consumption does not change.

**Proof:** Applying Cramer's rule on (10) yields

$$\begin{aligned} \frac{d\mu}{d\Omega} &= -\frac{\partial \Gamma_1}{\partial m} \frac{\partial \Gamma_2}{\partial x} \frac{\partial \Gamma_3}{\partial \Omega} / \Delta > 0, \\ \frac{dm}{d\Omega} &= \frac{\partial \Gamma_1}{\partial \mu} \frac{\partial \Gamma_2}{\partial x} \frac{\partial \Gamma_3}{\partial \Omega} / \Delta > 0, \\ \frac{dx}{d\Omega} &= -\frac{\partial \Gamma_1}{\partial \mu} \frac{\partial \Gamma_2}{\partial m} \frac{\partial \Gamma_3}{\partial \Omega} / \Delta > 0, \end{aligned}$$

Partial differentiation of equation (8) yields  $\frac{dc}{d\Omega} = -\frac{\partial \Lambda / \partial \Omega}{\partial \Lambda / \partial c} = 0$ . ■

Proposition 2 can be understood as follows. First, equation (6a) is the first order condition for the ruler's optimal choice of incentives to soldiers. From this equation, when the level of external threats increases, the direct effects are that marginal benefit of providing incentives to soldiers increases while marginal cost does not change. Because equilibrium values of  $m$  and  $x$  depend on the level of external threats, when the level of external threats increases, there are also indirect effects through changes in  $m$  and  $x$ . Increases in  $m$  and increase in  $x$  work in opposite directions. When the direct effects dominate the indirect effects, the ruler chooses a higher level of incentive for soldiers when the level of external threats increases. Second, from equation (6b) which is the first order condition for the ruler's optimal choice of military spending, when the level of external threats increases, the direct effects are that marginal benefit of military spending increases while marginal cost does not change. There are also indirect effects through changes in  $\mu$  and  $x$ . Increase in  $\mu$  and increase in  $x$  work in opposite directions. When the direct effects dominate the indirect effects, the ruler chooses a higher level of military spending when the level of external threats increases. Third, from equation (6c) which is the first order condition for the ruler's optimal choice of adopting the county form, with an increase either in the level of incentives to soldiers or an increase in military spending, marginal benefit of adopting the county form increases while marginal cost decreases. For indirect effects, increases in  $\mu$  and  $m$  work in the same direction as the direct effects do. Thus, the ruler chooses a higher level of adopting the county form.

As shown in the Reforms of Shang Yang, total amount of usable land in a state can be increased by methods such as eliminating or reducing boundaries of different pieces of land under the well-field system. The following proposition studies the impact of a change in the amount of land on the ruler's choices.

Proposition 3: An increase in the amount of land leads the ruler to choose a higher level of incentives for soldiers and a higher military spending. The ruler's consumption also increases.

Proof: Applying Cramer's rule on (10) yields

$$\frac{d\mu}{dT} = -\frac{\partial \Gamma_1}{\partial m} \frac{\partial \Gamma_2}{\partial x} \frac{\partial \Gamma_3}{\partial T} / \Delta > 0,$$

$$\frac{dm}{dT} = \frac{\partial \Gamma_1}{\partial \mu} \frac{\partial \Gamma_2}{\partial x} \frac{\partial \Gamma_3}{\partial \Omega} / \Delta > 0,$$

Partial differentiation of equation (8) yields  $\frac{dc}{dT} = -\frac{\partial \Lambda / \partial T}{\partial \Lambda / \partial c} > 0$ . ■

Proposition 3 can be understood as follows. When the amount of land increases, the direct effects are that the marginal benefit of providing incentive to soldiers increases and marginal cost decreases. The indirect effect through a change in the level of adopting the county form is ambiguous. When the direct effects dominate, the ruler chooses a higher level of incentives for a soldier. The impact of a higher amount of land on the ruler's choice of the degree of adopting the county form is ambiguous.<sup>18</sup>

Population can grow either through domestic birth or by immigration of foreigners. During the Reforms of Shang Yang, incentives were provided for immigration. The following proposition studies the impact of a change in population size on the ruler's choices.

Proposition 4: An increase in population size leads the ruler to choose a lower level of incentives for soldiers. The ruler's consumption increases.

Proof: Partial differentiation of (9a) and (9c) yields  $\frac{\partial \Gamma_1}{\partial L} \frac{\partial \Gamma_3}{\partial m} - \frac{\partial \Gamma_1}{\partial m} \frac{\partial \Gamma_3}{\partial L} > 0$ . Applying Cramer's rule on (10) yields

$$\frac{d\mu}{dL} = \frac{\partial \Gamma_2}{\partial x} \left( \frac{\partial \Gamma_1}{\partial L} \frac{\partial \Gamma_3}{\partial m} - \frac{\partial \Gamma_1}{\partial m} \frac{\partial \Gamma_3}{\partial L} \right) / \Delta < 0.$$

Partial differentiation of equation (8) yields  $\frac{dc}{dL} = -\frac{\partial \Lambda / \partial L}{\partial \Lambda / \partial c} > 0$ . ■

Proposition 4 can be understood as follows. An increase in population size changes both the marginal cost and marginal benefit of providing incentives to soldiers. In this model, the effect from the marginal cost change dominates. Thus, with a higher population size, the ruler chooses a lower level of incentives for soldiers. The impact of a higher population size on the level of military spending is ambiguous.

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<sup>18</sup> From (6c), when the amount of land increases, the direct effects on the choice of the county form are that marginal benefit of adopting the county form decreases while the marginal cost increases. However, there are indirect effects from changes in  $\mu$  and  $m$ . An increase in  $\mu$  or  $m$  increases the marginal benefit of adopting the county form and reduces the marginal cost. Since it is not clear whether the direct effects or indirect effects dominate, the impact of an increase in the amount of land on the adoption of the county form is ambiguous.



In China's history, standardization of language and measurement units helped to maintain China's unification. When transportation costs decrease, the cost of standardization will be lower. A higher level of standardization can reduce the level of variance in evaluating performance of soldiers. The following proposition studies the impact of a change in the level of variance on the ruler's choices.

Proposition 5: A higher level of variance leads the ruler to choose a lower level of effort for soldiers, a higher military spending, and a higher level of adopting the county form. The ruler's consumption does not change.

Proof: Partial differentiation of equation (9a) and (9c) yields  $\frac{\partial \Gamma_1}{\partial \sigma^2} \frac{\partial \Gamma_3}{\partial m} - \frac{\partial \Gamma_1}{\partial m} \frac{\partial \Gamma_3}{\partial \sigma^2} > 0$ . Applying Cramer's rule on (10) yields

$$\begin{aligned}\frac{d\mu}{d\sigma^2} &= \frac{\partial \Gamma_2}{\partial x} \left( \frac{\partial \Gamma_1}{\partial \sigma^2} \frac{\partial \Gamma_3}{\partial m} - \frac{\partial \Gamma_1}{\partial m} \frac{\partial \Gamma_3}{\partial \sigma^2} \right) / \Delta < 0, \\ \frac{dm}{d\sigma^2} &= \frac{\partial \Gamma_1}{\partial \mu} \frac{\partial \Gamma_2}{\partial x} \frac{\partial \Gamma_3}{\partial \sigma^2} / \Delta > 0, \\ \frac{dx}{d\sigma^2} &= -\frac{\partial \Gamma_1}{\partial \mu} \frac{\partial \Gamma_2}{\partial m} \frac{\partial \Gamma_3}{\partial \sigma^2} / \Delta > 0.\end{aligned}$$

Partial differentiation of equation (8) yields  $\frac{dc}{d\sigma^2} = -\frac{\partial \Lambda / \partial \sigma^2}{\partial \Lambda / \partial c} = 0$ . ■

Proposition 5 can be understood as follows. First, from equation (6a) which is the first order condition for the ruler's optimal choice of incentives for soldiers, when the level of variance increases, the direct effects are that marginal benefit of providing incentives to soldiers decreases while marginal cost increases. For indirect effects, an increase in  $m$  and an increase in  $x$  work in opposite directions. When the direct effects dominate the indirect effects, the ruler chooses a lower level of incentives for soldiers when the level of variance increases. Second, from equation (6c) which is the first order condition for the ruler's optimal choice of adopting the county form, when the level of variance increases, the direct effects are that marginal benefit of adopting the county form increases while marginal cost decreases. For indirect effects, a decrease in  $\mu$  and an increase in  $x$  work in opposite directions. When the direct effects dominate, the ruler chooses a higher level of adopting the county form when the level of variance increases.

Feudalism was adopted by the Zhou ruling house when it was difficult to manage a large country directly. Establishing counties means power will be concentrated in the central

government and this centralization of power could lead to oppositions and rebellions of nobles. Compared with other states, a state like Qin with limited influence of Confucianism will have a lower cost in establishing counties (Lin, 1981). A state with a lower influence of Confucianism can be captured by a lower value of the parameter  $a$ . The following proposition studies the impact of a change in the influence of nobles on the ruler's choices.

Proposition 6: When clans are stronger, the ruler chooses a lower level of incentives for soldiers, a lower military spending, and a lower level of the county form. The ruler's consumption decreases.

Proof: Applying Cramer's rule on (10) yields

$$\begin{aligned}\frac{d\mu}{da} &= -\frac{\partial \Gamma_1}{\partial m} \frac{\partial \Gamma_2}{\partial x} \frac{\partial \Gamma_3}{\partial a} / \Delta < 0, \\ \frac{dm}{da} &= \frac{\partial \Gamma_1}{\partial \mu} \frac{\partial \Gamma_2}{\partial x} \frac{\partial \Gamma_3}{\partial a} / \Delta < 0, \\ \frac{dx}{da} &= \frac{\partial \Gamma_1}{\partial \mu} \left( \frac{\partial \Gamma_2}{\partial a} \frac{\partial \Gamma_3}{\partial m} - \frac{\partial \Gamma_2}{\partial m} \frac{\partial \Gamma_3}{\partial a} \right) / \Delta < 0.\end{aligned}$$

Partial differentiation of equation (8) yields  $\frac{dc}{da} = -\frac{\partial \Lambda / \partial a}{\partial \Lambda / \partial c} < 0$ . ■

Proposition 6 can be understood as follows. When noble power increases, the direct effects are that marginal cost of adopting the county form increases while marginal benefit does not change. For indirect effects, a decrease in  $\mu$  and a decrease in  $m$  work in the same direction as the direct effects. Thus, the ruler chooses a lower level of county form. Proposition 6 can be used to compare the state of Qin with other states. Qin did not award land to nobles and the Reforms of Shang Yang were possible because influence of nobles was smaller in Qin. Proposition 6 can be used to interpret different results between Qin and Chu discussed in Section 2.

The percentage of soldiers in the population can be changed by modifying the age of military service. For example, when the minimum age for military service is decreased from eighteen to sixteen while the maximum age for military service does not change, then the percentage of soldiers in the population increases.<sup>19</sup> The following proposition studies the impact of a change in the percentage of soldiers on the ruler's choices.

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<sup>19</sup> In a military emergency such as the Changping Battle, the two involved states Qin and Zhao, eventually required almost all able-bodied individuals to participate in the war.

Proposition 7: An increase in the percentage of soldiers leads the ruler to choose a lower level of incentives for soldiers and a lower level of military spending. The ruler's consumption decreases.

Proof: Applying Cramer's rule on (10) yields

$$\frac{d\mu}{db} = \frac{\partial \Gamma_2}{\partial x} \left( \frac{\partial \Gamma_1}{\partial b} \frac{\partial \Gamma_3}{\partial m} - \frac{\partial \Gamma_1}{\partial m} \frac{\partial \Gamma_3}{\partial b} \right) / \Delta < 0,$$

$$\frac{dm}{db} = \frac{\partial \Gamma_1}{\partial \mu} \frac{\partial \Gamma_2}{\partial x} \frac{\partial \Gamma_3}{\partial b} / \Delta < 0.$$

Partial differentiation of equation (8) yields  $\frac{dc}{db} = -\frac{\partial \Lambda / \partial b}{\partial \Lambda / \partial c} < 0$ . ■

Proposition 7 can be used to interpret different results between Qin and Wei discussed in Section 2. Proposition 7 can be understood as follows. First, from equation (6a) which is the first order condition for the ruler's optimal choice of incentives for soldiers, when the percentage of soldiers increases, the direct effects are that marginal benefit of providing incentives to soldiers decreases while marginal cost increases. The indirect effect through the level of adopting the county form is ambiguous. When the direct effects dominate, the ruler chooses a lower level of incentives for soldiers when the percentage of soldiers in the population increases. Second, from equation (6b) which is the first order condition for the ruler's optimal choice of military spending, when the percentage of soldiers increases, the direct effects are marginal benefit of military spending decreases while marginal cost increases. The indirect effect through the level of adopting the county form is ambiguous. When the direct effects dominate, the ruler chooses a lower level of military spending when the percentage of soldiers in the population increases. The impact of a change in the percentage of soldiers on the ruler's choice of the degree of adopting the county form is ambiguous.<sup>20</sup>

Rather than being a soldier, an individual could engage in commerce. When commerce was discouraged as in the Reforms of Shang Yang, this could be captured by a decrease in the

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<sup>20</sup> From (6c), when the percentage of soldiers increases, the direct effects are that marginal benefit of adopting the county form increases while the marginal cost decreases. However, there are indirect effects from changes in  $\mu$  and  $m$ . A decrease in  $\mu$  or  $m$  decreases the marginal benefit of adopting the county form and increases the marginal cost. Since it is not clear whether the direct effects or indirect effects dominate, the impact of an increase in the percentage of soldiers on the adoption of the county form is ambiguous.

reservation utility of a soldier. The following proposition studies how a change in reservation utility affects the ruler's choices.

Proposition 8: An increase in the level of reservation utility of a soldier changes neither military spending nor the level of incentive provision to soldiers. The equilibrium level of county adoption increases. The ruler's consumption does not change.

Applying Cramer's rule on (10) yields

$$\begin{aligned}\frac{d\mu}{dR} &= 0, \\ \frac{dm}{dR} &= 0, \\ \frac{dx}{dR} &= \frac{\partial \Gamma_1}{\partial \mu} \frac{\partial \Gamma_2}{\partial R} \frac{\partial \Gamma_3}{\partial m} / \Delta > 0.\end{aligned}$$

Partial differentiation of equation (8) yields  $\frac{dc}{dR} = -\frac{\partial \Lambda / \partial R}{\partial \Lambda / \partial c} = 0$ . ■

Proposition 8 can be understood as follows. From equation (6c) which is the first order condition for the ruler's optimal choice of the degree of adopting the county form, the direct effects of an increase in reservation utility of a soldier are that marginal benefit of adopting the county form increases while marginal cost decreases. Since  $\mu$  and  $m$  do not change with the level of reservation utility, the indirect effect is zero. Thus, the ruler chooses a higher level of adopting the county form in equilibrium.

Faced with frequent nomadic invasion from the west side of the state, Qin people could be more militarily oriented compared with other states. That is, compared with soldiers in other states, Qin soldiers could be less risk averse. The following proposition studies the impact of a change in the degree of risk aversion on the ruler's choices.

Proposition 9: When soldiers are more risk averse, the ruler chooses a higher level of incentives for soldiers, a higher level of military spending, a higher level of adopting the county form. The ruler's consumption does not change.

Proof: Applying Cramer's rule on (10) yields

$$\begin{aligned}\frac{d\mu}{dr} &= \frac{\partial \Gamma_1}{\partial m} \frac{\partial \Gamma_2}{\partial x} \frac{\partial \Gamma_3}{\partial r} / \Delta > 0, \\ \frac{dm}{dr} &= \frac{\partial \Gamma_1}{\partial \mu} \frac{\partial \Gamma_2}{\partial x} \frac{\partial \Gamma_3}{\partial r} / \Delta > 0,\end{aligned}$$

$$\frac{dx}{dr} = -\frac{\partial \Gamma_1}{\partial \mu} \frac{\partial \Gamma_2}{\partial m} \frac{\partial \Gamma_3}{\partial r} / \Delta > 0.$$

Partial differentiation of equation (8) yields  $\frac{dc}{dr} = -\frac{\partial \Lambda / \partial r}{\partial \Lambda / \partial c} = 0$ . ■

Similar to the case of an increase in the degree of risk aversion of soldiers, when the cost of effort for a soldier increases, it can be shown that the ruler chooses a lower level of incentives to soldiers. Also, the ruler chooses a higher level of military spending.

## 5. Conclusion

The Reforms of Shang Yang is a significant event in China's history. Shang Yang took measures to rationalize tax, military, and government administration and his measures made the Qin economy more efficient in fighting wars. Issues reflected in the Reforms such as institutional complementarities and impact of culture on the choice of institutions are still relevant today.

In this paper, we have studied how various factors such as the level of external threats and population size affect the adoption of institutions in the Reforms of Shang Yang in a mathematical model. We have established the following results analytically. First, different reform measures such as providing strong incentives for soldiers and the adoption of the county form are complementary. Second, an increase in the level of external threats or the degree of risk aversion of soldiers induces the ruler to choose stronger incentives for soldiers, a higher degree of military spending, and a higher level of adopting the county form. However, the ruler's consumption does not change. Third, a higher population leads the ruler to choose a lower incentive for soldiers and the ruler's consumption increases. Finally, the ruler chooses a lower level of consumption, a lower level of incentive for soldiers, a lower military spending, and a lower level of adopting the county form when clans are stronger.

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