Unequal Recovery from Recessions: Skills Learning Among Young Workers

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Abstract

Numerous studies indicate that graduating and entering the job market during a recession leads to persistent wage losses, initially at 2% per one-point rise in the national unemployment rate and remaining significant over a decade. However, this statistic conceals significant heterogeneity in outcomes. I analyze the National Longitudinal Survey of Youth data set and use quantile regression to summarize the variation of young workers' life-cycle wage profile. I find workers in higher residual wage quantiles have faster recovery speed. This heterogeneity can be attributed to unobservable workers' characteristics. To understand these findings, I present a life-cycle model with two-dimensional Ben-Porath skills accumulation among workers differing in three dimensions. Dimensions related to wage determination but not directly observed from data: true abilities, initial self-beliefs about true abilities, and initial stock of skills. This model extends the classical Ben-Porath model by accounting for workers' uncertainties about their skills accumulation and introducing aggregate fluctuations in the demand for skills. The model addresses the inefficiency of the recovery process, particularly how recessions alter time investment in skills and career paths, especially for those with biased self-beliefs about their true abilities. In an economy with more precise signals, the recovery process is longer. Increased investments in skill accumulation due to more information friction serve as a buffer when a recession hits. The counterfactual experiment reveals that the endogenous change in workers' investment in skills accumulation due to the recession explains up to 20% of wage losses every year throughout the recovery period and 90% from the recovery periods after the recession. Finally, I find that unequal recovery among workers primarily links to differences in true skill accumulation abilities rather than differences in initial skill levels or biased self-beliefs.

Keyword: Scarring Effect; Unequal Recovery; Multidimensional Skills; Learning

JEL Classification Code: E24, J24, J31, J41, J62.

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1 Introduction

Extensive empirical research reveals a striking fact: graduating and initiating careers during a recession leads to persistent adverse effects on workers' wage trajectories throughout their lives. This adverse impact gradually diminishes over time, with affected workers eventually reaching the wage profile as if they entered the labor market during non-recessionary periods (referred to as the recovery process). This long-term negative effect of entering the labor market during a recession on wages, the so-called "scarring effect", is also heterogeneous across workers with different characteristics, such as educational levels and cohorts.[1] So, why does a short-run recession have long-term impacts on the wages of young workers? Do individuals with similar observable characteristics experience the same wage recovery paths throughout their careers when they enter the job market during the same recession? If not, what is the key factor driving these variations?

Addressing these questions holds significant implications, particularly for predicting the life-cycle wage losses of heterogeneous young workers who graduated and entered the job market during recent recessions, such as the COVID-19 pandemic. Furthermore, comprehending the primary causes of the persistent effect and associated heterogeneity is instrumental in identifying potential policy interventions to mitigate labor market disruptions caused by recessions.

I focus on high school graduates because they experience a more significant long-term impact when graduating and entering the labor market during a recession, as compared to college graduates. Using data from the National Longitudinal Survey of Youth, I estimate the effects of changes in the national unemployment rate at the time of graduation and entering the labor market on the lifetime log wage of high school graduates who get full-time jobs in the recession. The findings reveal that, on average, high school graduates who enter the labor market during a recession experience a 4% decline in hourly wages for every one-percentage-point increase in the unemployment rate at the time of graduation and entry. This effect persists but gradually diminishes over a 15-year period. The magnitudes of wage losses and recovery rates exhibit a linear relationship with changes in the unemployment rate. These findings suggest that young high school graduates may face an initial 20% wage reduction for a 5-percentage-point increase in the unemployment rate during the COVID-19 pandemic. This effect remains persistent over the decades. Despite controlling for observable differences in worker characteristics, quantile regression results indicate significant variation in recovery speeds after a recession across different residual wage quantiles. Specifically, I find that workers in higher residual wage quantiles exhibit faster recovery speeds. This paper is the first to empirically document the existence of heterogeneity in the recovery process, even after accounting for differences in directly observable characteristics. These two findings have inspired me to build a model to further explore the reason for the scarring effect and the causes for the heterogeneity.

The main objective of this paper is to elucidate the causes behind the wage recovery

[1] For example, see Kahn (2010) and Speer (2016) for discussions within the United States, Oreopoulos et al. (2012) for Canada, Liu et al. (2016) for Norway, and Cockx and Ghirelli (2016) for Belgium. Von Wachter and Bender (2008), Schwandt and Von Wachter (2019), Umkehrer (2019), Wachter (2020), and Rinz (2022) document the heterogeneity in the effects across workers' characteristics such as education and cohort.

process and the primary sources of heterogeneity within it. I view individuals' life-cycle wages as influenced by two primary factors: the evolution of workers' levels of skills and how firms value workers' skills over time. The prior literature does not quantify the impact of changes in investment on skills accumulation on wage recovery for young workers. Moreover, variations in the wage recovery process among workers are linked to differences in decisions about skills accumulation. Apart from differences in inherent characteristics, such as true skill accumulation abilities, I also consider differences in how workers perceive their true skill accumulation abilities, which no one incorporates in the scarring effect discussion. This informational friction is crucial for young workers making skills investment decisions, given their limited understanding of their abilities at a young age, explaining the declining probability of switching occupations by age. Hence, in this paper, I investigate three questions for young workers who get full-time jobs during a recession: (1) how much of the lifetime wage loss is explained by the decline in investment in skills accumulation due to the recession?; (2) what role do imprecise signals received by workers about true abilities in skills investment play in the recovery process?; and (3) what factors drive the unequal recovery process from recessions among young workers in different residual wage quantiles?

To answer these questions, I extend the multidimensional skills accumulation model with endogenous occupational choices in Cavounidis and Lang (2020) by introducing two information frictions: workers' idiosyncratic learning about true abilities in the spirit of Guvenen et al. (2020), and persistent aggregate shocks to skill productivity. In the paper, I simulate a recession by dropping skills' productivity by a certain percentage in the entry year to match the initial percentage decline in wages for high school graduates for any particular recession. As in the standard Ben-Porath model, workers face trade-offs in current and future consumption: investing more time in skill accumulation increases future consumption but decreases current consumption due to reduced working hours. Additionally, the empirical findings underscore the importance of unobserved characteristics in explaining the heterogeneity in recovery speeds among young workers. Thus, I consider workers differ in three initial conditions that influence these trade-offs and, consequently, the long-term wage profile: true abilities on skill accumulation, initial signals about true abilities, and initial stock of skills. With idiosyncratic learning processes on true abilities on skill accumulation and endogenous occupational choices, the model is relevant for understanding the potential causes of heterogeneity in the scarring effect. For instance, workers with higher true abilities in skill accumulation experience lower recovery costs, leading to faster recovery. Moreover, recovery processes are inefficient because the decline in the value of skills may lead workers to work in the wrong occupation, especially for some workers with biased self-belief about themselves.

My model generates simulations consistent with salient empirical findings in both life-cycle wage and life-cycle occupational switching patterns. I find the scarring effect lasts even longer on average in an economy where workers have more precise signals about their true skill accumulation abilities due to less "investment motives." I consider two reasons for persistent lower wages following a recession in the model. One is that the economy does not value skills as usual in the first several years after the recession. Additionally, workers are reluctant to invest as much time in skills as during the non-recessionary scenarios, constituting the second reason for the long-term effect. I use a counterfactual experiment to decompose the contributions of each reason to the lifetime wage losses due to the recession. I quantitatively determine that, while the recession is primarily driven in my model by the decline in aggregate skills' productivity, the endogenous decline in skills accumulation choices still contributes substantially to the lifetime wage loss. While it accounts for up to 20% of the annual wage losses in present value throughout the recovery period, it accounts for a staggering 90% of the losses from the 8th year to the 15th year after the recession. Finally, the unequal recovery process among workers can be primarily explained by the differences in true skill accumulation abilities, but not by the differences in initial skill levels or biased self-beliefs.

The paper proceeds as follows. Section 2 lists the related literature and the marginal contributions of the paper. Section 3 describes data sets and empirical findings in this paper. Section 4 presents our detailed model setup and implications. Section 5 illustrates calibration strategies and results from the counterfactual analyses. The conclusion is drawn in Section 6.

2 Literature Review

This paper contributes to the "scarring effect" literature studying the long-term effect of recession on workers' wages throughout their lives.[2] There is a consensus that the adverse effect of exposure to recession is persistent.[3] According to Schwandt and Von Wachter (2019), who combine several data sets with extensive coverage over time and diverse workers' characteristics, an increase of one percentage point in the unemployment rate leads to a 2% decline in entry-level wages on average, and this reduction remains statistically significant even a decade after graduation. Moreover, initial wage losses positively correlate with the unemployment rate, meaning that more severe recessions, like the early 1980s recession and the latest COVID-19 recession, can result in more considerable initial wage reductions and longer recovery periods (Kahn (2010)). In addition to the mean analysis, researchers are also interested in the heterogeneity of the scarring effect across different groups of workers.

Previous studies have shown that workers of different ages when the recession occurs have different recovery processes, commonly known as the cohort effect. Yagan (2019) notes that the effects on employment are more pronounced for older workers, with less noticeable variations in earnings across different age groups. The study also reveals that lower-earning groups witnessed a more substantial decline, implying that the Great Recession heightened inequality among workers with varying initial skill levels. In line with Yagan (2019), Rinz (2022) utilizes administrative data and expands the analysis to include

[2]In this paper, my analysis mainly focuses on wages. Thus, it is a discussion conditional on individuals who get full-time jobs when they enter the labor market in a recession. The previous literature often focuses on displaced workers rather than young workers who get jobs during recessions and are stable in the labor force. Displaced workers are defined as those who involuntarily lose their jobs due to reasons beyond their control, such as company closures during recessions. Refer to Davis and Von Wachter (2011), Huckfeldt (2022), and Schmieder et al. (2023) for discussions on the scarring effect experienced by displaced workers during recessions.

[3]Wachter (2020) summarizes several main findings that the current literature agrees on.

more cohorts. He finds more persistent and relatively more considerable adverse earnings effects among younger workers. Moreover, similar to Schwandt and Von Wachter (2019), the magnitude of earnings effects increases roughly linearly with the severity of the recession.

In this paper, I only focus on young workers to eliminate the discussion regarding cohort effects. This choice is motivated by two factors that make young workers an ideal cohort for investigating the scarring effect. Firstly, they have longer remaining lifetimes, making them more tractable for recessions' more profound and extended impacts. Secondly, their limited financial wealth compared to their labor income allows me to disregard wealth as a major influencing factor when considering the primary mechanism behind the scarring effect (refer to Glover et al. (2020) for relevant evidence).

Except for cohort differences, the heterogeneity of the recovery process is also mostly evident across groups of workers categorized by educational levels. Most of the current empirical analyses study young workers with higher education (e.g., Oyer (2006) for economics PhDs, Kahn (2010) and Altonji et al. (2016) for college graduates in the United States, and Oreopoulos et al. (2012) for college graduates in Canada). For young college graduates, the initial wage drop is approximately 10% for a recession with a rise of 4-5 percentage points in the unemployment rate, and the effect fades after about ten years. Besides, they may end up entering different occupations or otherwise less appealing jobs. There are only a few papers discussing less-educated workers. Genda et al. (2010) estimates the scarring effects for workers that complete high school or less in Japan and the United States. They find persistent negative effects on earnings of the unemployment rate at graduation for less-educated workers in Japan but temporary effects for workers in the United States. However, their analyses use the potential graduation year instead of the actual information. Speer (2016) uses NLSY's weekly work history and finds temporary effects on earnings for workers with 9-12 years of education in the United States. However, Schwandt and Von Wachter (2019) and Wachter (2020) use more extensive data sets and find that the effect for high-school graduates is about double the effect for college graduates and more persistent. Additionally, there are no significant variations in the impact of adverse labor market entry conditions among workers of different genders and races. In this paper, I only target high school graduates and find similar results as Schwandt and Von Wachter (2019).

To my best knowledge, this is the first paper to address heterogeneity in recovery processes among workers in different residual wage quantiles. Additionally, the existing literature neither quantifies the impact of changes in skill accumulation investment on wage recovery for young workers nor explains the primary reasons for unequal recovery. This paper fills in these gaps. Furthermore, this study introduces an important informational friction for young workers into the model by incorporating the idiosyncratic learning process related to workers' true skill accumulation abilities, a novel addition to understanding the scarring effect on life-cycle wages.

In this paper, I introduce an extended human capital theory to elucidate the primary cause behind the heterogeneity in the scarring effect, an approach that has not been undertaken previously. Entering the labor market in a recession has lasting impacts on individuals' occupation choices (Oyer (2006)). Young workers facing bad economic conditions often find themselves in less appealing job positions (Kahn (2010), Wachter (2020)).

Additionally, Oreopoulos et al. (2012) demonstrates that individuals projected to have high earnings based on factors such as college, major, and degree type tend to experience better outcomes during recessions, with only short-term setbacks. On the other hand, individuals at the bottom of the predicted earnings distribution can experience permanent reductions in earnings. The root cause of these outcomes, whether stemming from shifts in skill investment or unfavorable signaling, remains unclear. Thus, I introduce endogenous occupation choices and learning processes about true abilities to the model.

The model closely resembles those described in the Guvenen et al. (2020) and the Cavounidis and Lang (2020). While Guvenen et al. (2020) constructs a multidimensional skills accumulation model with endogenous occupational choices, addressing the impact of skills mismatch on workers' earnings profiles, it does not consider occupational choices and wages influenced by aggregate economic conditions like recessions. Additionally, its skill accumulation process removes the standard Ben-Porath model's trade-off between present and future consumption, a key focus in my paper. Cavounidis and Lang (2020) introduces a Ben-Porath style multidimensional skills accumulation model with endogenous occupational choices, exploring workers' age-related occupational responses to MIT shocks through simulations. My model is akin to Cavounidis and Lang (2020), integrating workers' idiosyncratic learning about true abilities from Guvenen et al. (2020), and includes persistent aggregate shocks to skill productivity, making it relevant for the scarring effect discussion.

3 Data

The primary data set used in this paper is the National Longitude Survey of Youth for the 1979 Cohort (NLSY79). NLSY79 tracks the lives of a nationally representative sample of young Americans born between 1957 and 1964, which provides detailed workers' characteristics,[4] educational records, employment history, and occupational titles. The interviews were conducted annually from 1979 to 1994, after which the frequency became biennial. I utilize the panel data spanning from 1979 to 2018. There are two reasons why the NLSY79 data set is well-suited for this paper. The first reason is that the data set tracks the same groups of workers' labor market outcomes for a long time after graduating during the early 1980s recession, which makes it a perfect sample for me to observe the long-term consequences of the "scarring effect." In addition, the data set also contains some variables that measure individual skill levels, such as the Armed Services Vocational Aptitude Battery (ASVAB) test scores. Prior to the first survey, about 94% of participants completed the tests.[5] In this paper, I consider the test scores as signals to workers' true abilities in accumulating different skills. I also obtain the real hourly wage and workers'

[4]Worker's characteristics contains information such as age, gender, and Armed Services Vocational Aptitude Battery (ASVAB) test scores.

[5]The ASVAB consists of ten tests to measure an individual's skills and knowledge. These tests include: (1) general science, (2) arithmetic reasoning, (3) word knowledge, (4) paragraph composition, (5) numerical operations, (6) coding speed, (7) auto and shop information, (8) mathematics knowledge, (9) mechanical comprehension, and (10) electronics information. I compute the accuracy rates in each ASVAB category for each individual and use them to construct the prior beliefs about true skill accumulation abilities.

occupations of each individual for a lengthy coverage of time.

3.1 Sample Selection

I construct the yearly panels from the NLSY79's Work History Data File from 1979 to 2018. The detailed process to link weekly data to annually is similar to Guvenen et al. (2020). Firstly, following Neal (1999), Pavan (2011) and Guvenen et al. (2020), I define the main job as the one with the most prolonged working hours in each given year. Then, I keep only the record of the main job and merge it with other yearly reported workers' demographic information and detailed employment information, such as occupation and hourly wage. In the paper, I aim to estimate the impact of high school graduates' wage losses on graduating and entering the labor market during a recession. Some people argue that young graduates may pursue higher degrees to avoid the negative impact of the recession. To avoid this possibility, I only focus on workers who enter the labor market immediately after receiving degrees and, at the same time, are strongly attached to the labor market. That is to say, the new graduates immediately get full-time jobs (i.e., working hours for more than 1200) after graduation for at least two years. Additionally, they do not further obtain any higher degree. As documented in the previous literature, graduating in the labor market during a recession has persistent effects on workers' occupation choices. Thus, I drop the military sample in this paper because those have more restrictions and rarely change occupations. I ensure that individuals in the sample have valid information about the highest degree and year completion. Using the personal consumption expenditure (PCE) deflator, I convert nominal to real hourly wage, taking 2000 as the base year. Then, I drop observations if the real hourly wage is below \$1 or larger than \$1,000. I also drop those observations without occupational and industry codes. Then, I only focus on employed individuals with valid wage information in the year of graduation. Finally, I restricted the sample to only workers with high school degrees. The complete description of our sample processing procedures is illustrated in Appendix A.1. The final sample has $24,199$ worker-year observations, containing $1,053$ high school graduates working across 257 occupations from 1979 to 2018.

3.2 Sample Descriptions

Table 1 displays the descriptive statistics for the sample. On average, labor market outcomes for workers can be observed for over 20 years. Moreover, over half of the participants in the sample are non-Hispanic and non-Black individuals. To understand the wage losses resulting from graduating and entering the labor market during a recession, I need to compare the labor market performance of workers who enter before, during, and after recessions. Since the early 1980s recession occurred during 1981-1982, I categorize workers who attained a high school degree and began working between 1979 and 1980 as entering before the 1980s recession. Those who graduated and entered the labor market during 1981-1982 are classified as entering during the 1980s recession. Workers who graduated and entered after 1982 constitute the "Entry post-1980s recession" group. The distribution of workers among the entry periods group is nearly even in the sample. Although the sample contains more males (around 60%) than females, the composition of workers with the above characteristics in the two genders is similar.

Statistics	All Sample	Female	Male
Number of Individuals	1,053	428	625
Number of Observations	24,199	9,112	15,087
Compositions of Unique Workers			
Hispanic	18.90%	17.52%	19.84%
Black	21.27%	18.69%	23.04%
Entry Pre-1980s Recession	35.80%	37.38%	34.72%
Entry During 1980s Recession	33.52%	33.64%	33.44%
Entry Post-1980s Recession	30.68%	28.98%	31.84%

Table 1: Descriptive Statistics of the Sample in NYLS79, 1979-2018

In Table 2, the distribution of workers in my sample is presented based on their entry occupational and industrial groups. Occupational codes have been converted into 1990 3-digit census codes and categorized into 6 major groups as used by IPUMS.[6] For industrial codes, I have converted them to 1970 1-digit census codes, resulting in 12 industrial categories. In this sample, a significant proportion of workers are employed in technical sales and administrative support occupations, as well as operators, fabricators, and laborers. Approximately 19% of workers have service occupations as their initial jobs, while only a small proportion are engaged in managerial and professional specialty occupations. The majority of workers are found in the wholesale and retail trade industries. Also, about 20% of workers are in manufacturing industries.

Table 3 displays the average log hourly wage for high school graduates when they enter the labor market before, during, and after the recession. The table illustrates that high school graduates who enter the labor market before and after the recession tend to get the first jobs with higher hourly wages compared to those who enter during the recession. This pattern holds true across different genders.

3.3 The Average Scarring Effect Across Workers

This section presents the impact on wages over the life cycle for high school graduates who graduated and entered the labor market in a recession, using predictions based on results obtained from the early 1980s recession. For the baseline specification, I adopt the empirical design in Kahn (2010), which uses the national unemployment rate as an

[6] *Managerial and Professional Specialty Occupations* (*codes* $3-200$); *Technical Sales and Administrative Support Occupations* (*codes* $200-430$); *Service Occupations* (*codes* $430-470$); *Farming, Forestry, and Fishing Occupations* (*codes* $470-500$); *Precision Production, Craft, and Repair Occupations* (*codes* $500-700$); *Operators, Fabricators, and Laborers* (*codes* $700-900$). Check https://usa.ipums.org/usa/volii/occ1990.shtml for more details about characterizations.

Occupational Group	% of Workers	Industrial Group	% of Workers
Managerial and Professional Specialty Occupations	6.36%	*Agriculture, Foresty, & Fisheries*	3.80%
Technical Sales and Administrative Support Occupations	30.01%	*Mining*	0.47%
Service Occupations	19.37%	*Construction*	7.22%
Farming, Forestry, and Fishing Occupations	4.18%	*Manufacturing*	20.13%
Precision Production, Craft, and Repair Occupations	13.49%	*Transportation, Communications & Other Public Utilities*	3.61%
Operators, Fabricators, and Laborers	26.59%	*Wholesale & Retail trade*	36.37%
		Finance, Insurance & Real Estate	4.56%
		Business & Repair Services	5.79%
		Personal Services	2.94%
		Entertainment & Recreation Services	1.80%
		Professional & Related Services	10.64%
		Public Administration	2.66%

Table 2: Distribution of Workers in Entry Occupations & Industries

Entry Log Wages	All Sample	Female	Male
Entry Pre-1980s Recession	2.02	1.97	2.05
Entry During 1980s Recession	1.88	1.82	1.93
Entry Post-1980s Recession	2.05	1.93	2.12

Table 3: Entry Log Wages

indicator of the economic condition and assumes the impact is linear.[7] Table 1 presents the annual national unemployment rates from 1980 to 1990, and we can observe substantial variation in the national unemployment during this period, during which almost all the sample graduated. The unemployment rate increased by about 2.5 percentage points during the recession.

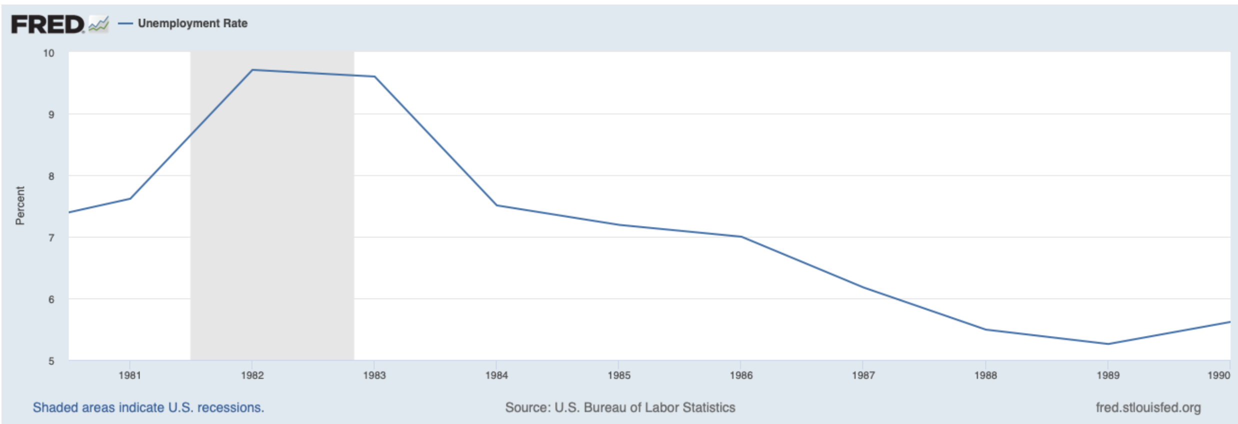


Figure 1: Annual National Unemployment Rates from 1980 to 1990

I first estimate the following standard Mincer-style regression augmented with unemployment rate variables to observe the scarring effect on average across workers,

$$\ln(wage_{it}) = \beta_0 + \beta_1 UE_i + \beta_2 UE_i \times Exp_{it} + \beta_3 Exp_{it} + \beta_4 Exp_{it}^2 + \boldsymbol{\gamma}^{'}\boldsymbol{Y}_t + \boldsymbol{\Psi}^{'}\boldsymbol{IA}_i + u_{it}, \quad (1)$$

where i and t represent indices for individuals and years, respectively. The dependent variable $\ln(wage_{it})$ is the log real hourly wage for individual i at year t. UE_i denotes the unemployment rate during the graduation year for individual i, so it is individual-specific. Additionally, Exp_{it} shows the number of years since high school graduation, referred to as potential experience. I use potential experience instead of actual labor market experience because the latter may be endogenously related to the UE_i. $\boldsymbol{Y}_t$ is a vector of contemporaneous year indicators, and $\boldsymbol{IA}_i$ is the vector of the graduation age indicators. By incorporating the entry-age dummies, I also eliminate the possible cohort effect. I denote the error term as u_{it}, and I cluster the standard errors by years of graduation in the regression.

The two critical relevant estimators for the scarring effect discussion are β_1 and β_2. β_1 measures the immediate percentage wage loss for a one percentage point increase in the unemployment rate upon entry, and β_2 measures how the marginal effect changes over time. Table 4 reports these long-term effects on real hourly wages. Column (1) summarizes the results from the regression equation (1). I also control for gender and race dummies (see columns (2)) to eliminate the potential scarring effect differences across

[7]Based on the analysis of the United States and Canada, Wachter (2020) indicates that the short- and long-term effects of cyclical economic conditions are roughly proportional to changes in the unemployment rate, demonstrating a constant recovery speed. I consider the changing recovery speed also, and the regression results are shown in Appendix A.2.

these two observed characteristics. Moreover, I incorporate control variables that are typically associated with workers' abilities, including the characteristics of their entry jobs and ASVAB test scores. Column (3) introduces controls for major occupational and industrial groups, while Column (4) presents results after incorporating workers' ASVAB test scores as additional controls.

	(1)	(2)	(3)	(4)
A: Regression coefficients				
UE	-0.0407***	-0.0388***	-0.0313***	-0.0286***
	[0.009]	[0.008]	[0.008]	[0.008]
$UE \times Exp$	0.0016***	0.0017***	0.0016***	0.0017***
	[0.000]	[0.000]	[0.000]	[0.000]
B: Fitted effects for selected years of experience				
1	-0.0391***	-0.0371***	-0.0297***	-0.0269***
	[0.009]	[0.008]	[0.008]	[0.008]
5	-0.0326***	-0.0304***	-0.0231***	-0.0202***
	[0.009]	[0.008]	[0.008]	[0.007]
10	-0.0244***	-0.0219***	-0.0149**	-0.0118*
	[0.008]	[0.007]	[0.007]	[0.007]
15	-0.0163**	-0.0135**	-0.0068	-0.0034
	[0.008]	[0.006]	[0.007]	[0.007]
Controls:				
Gender & Race		X	X	X
Entry Occupational & Industrial Groups			X	X
ASVAB Test Scores				X
Obs.	24199	24199	24199	24199
Adjusted R-squared	0.236	0.271	0.308	0.318

Clustered standard errors in brackets
* p<0.10, ** p<0.05, *** p<0.01

Table 4: OLS Regression Results

Panel A presents the coefficients corresponding to the unemployment rate in the graduation and entry year and its interaction with potential experience. Panel B illustrates the estimated impact of a one-percentage-point increase in the unemployment rate when individuals graduate and enter the labor market for specific years following the occurrence. I find the initial percentage loss in wage is around 4% for a one-percentage-point increase in the entry unemployment rate, statistically significant at the 1% level. This effect dissipates by 0.0016 log points, which I call the recovery speed. The result is similar as discussed in Schwandt and Von Wachter (2019). From Panel B, I observe young high school graduates begin to catch up, and the effect remains statistically significant at the 5% level even after 15 years. In the first decade, the effect remains substantial and statistically significant at the 1% level. These estimates show minimal change even after adjusting for gender and race (Column (2)). However, once I account for controls related to workers' abilities (Columns (3) and (4)), the initial wage loss percentage decreases, resulting in a shorter recovery period.

One might worry about the endogeneity problem discussed in the existing literature. That is, the timing of graduation and labor market entry are endogenously choices of individuals affected by the labor market conditions. I will argue it is not the case for high school graduates in my sample because, unlike college students who take a gap year and find an internship to work and then come back to school to graduate, wishing they get a better job when the economy condition gets better, it is unlikely for high school students to drop out and then continue to finish the degrees later. High school graduates do not have the incentive to delay their graduation. The other possible concern is that high school graduates may continue receiving higher education during the recession and drop out when the economy improves. However, I only keep those workers whose highest degree is high school, and they graduate and immediately get full-time jobs in the first few years. Moreover, they are unlikely to pay for the high college tuition to avoid going to the market in a recession.

3.4 The Heterogeneity in the Scarring Effect

Then, I investigate scarring effect heterogeneity across workers in various residual wage groups using quantile regression. In particular, I use quantile regression instead of segmenting the dependent variable based on its unconditional distribution and applying OLS to a subsample. This approach utilizes the entire sample and avoids sample selection issues, as advocated by Heckman (1979). The baseline equation for quantile regression remains the same as equation (1), but the estimation approach is different from OLS. Unlike OLS regression, which estimates conditional mean functions, quantile regression focuses on conditional quantile points for the distribution of the dependent variable. While OLS minimizes the sum of squared residuals, quantile regression estimators for residual quantile $\tau (0 < \tau < 1)$ minimize a weighted sum of absolute residuals ($Q(\tau)$), as given by the formula:

$$Q(\tau) = \sum_{it:u_{it} \geq 0} \tau |u_{it}| + \sum_{it:u_{it} < 0} (1 - \tau) |u_{it}|.$$

Table 5 summarizes the results. Similar to the previous section, different columns contain results under different control variables. Column (1) shows the results after controlling the individual-specific entry age and year dummies. Additionally, column (2) includes gender and race dummies, column (3) reports results after controlling the entry-job characteristics, and column (4) presents results after controlling the ASVAB test scores.

In Table 5, I only report the results of three quantiles: 25^{th}, 50^{th}, and 75^{th}. First, the scarring effect still exists for workers in all residual wage quantiles. Secondly, conditional on all possible individual heterogeneity in observed characteristics, I still find differences in recovery processes, which are unexplained in the existing literature. Moreover, there are no significant differences in the initial wage losses for workers under different quantiles. However, workers in higher residual quantiles have faster recovery speed, and the findings are robust under these four cases. Additionally, when I account for workers' ability indicators, such as entry job characteristics and test scores, I observe that workers in all residual wage quantiles experience shorter recovery periods. It implies the importance of some unobserved characteristics in explaining the heterogeneity in recovery speed. Thus, graduating and entering the labor market during the recession generally has less prolonged negative impacts on workers in higher wage residual quantiles.

The findings are even more evident if I plot the initial wage losses and the recovery

	(1)			(2)			(3)			(4)		
	Q(0.25)	Q(0.50)	Q(0.75)	Q(0.25)	Q(0.50)	Q(0.75)	Q(0.25)	Q(0.50)	Q(0.75)	Q(0.25)	Q(0.50)	Q(0.75)
A: Regression coefficients												
UE	-0.0285***	-0.0443***	-0.0405***	-0.0344***	-0.0436***	-0.0375***	-0.0284***	-0.0304***	-0.0325***	-0.0272***	-0.0246***	-0.0291**
	[0.009]	[0.010]	[0.012]	[0.010]	[0.007]	[0.011]	[0.010]	[0.007]	[0.011]	[0.010]	[0.008]	[0.011]
$UE \times Exp$	0.0007***	0.0019***	0.0026***	0.0014***	0.0018***	0.0024***	0.0012***	0.0017***	0.0022***	0.0012***	0.0015***	0.0021***
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.001]
B: Fitted effects for selected years of experience												
1	-0.0278***	-0.0423***	-0.0380***	-0.0331***	-0.0418***	-0.0351***	-0.0272***	-0.0286***	-0.0303***	-0.0261***	-0.0231***	-0.0269***
	[0.009]	[0.010]	[0.011]	[0.010]	[0.007]	[0.011]	[0.010]	[0.007]	[0.011]	[0.010]	[0.007]	[0.011]
5	-0.0249***	-0.0346***	-0.0278**	-0.0277***	-0.0348***	-0.0257***	-0.0226***	-0.0217***	-0.0215**	-0.0214***	-0.0172***	-0.0206**
	[0.009]	[0.010]	[0.011]	[0.010]	[0.007]	[0.010]	[0.009]	[0.006]	[0.010]	[0.009]	[0.007]	[0.010]
10	-0.0213**	-0.0250**	-0.0150	-0.0209**	-0.0260***	-0.0139*	-0.0168**	-0.0130**	-0.0104	-0.0156**	-0.0098	-0.0078
	[0.009]	[0.010]	[0.009]	[0.009]	[0.006]	[0.008]	[0.008]	[0.006]	[0.009]	[0.008]	[0.007]	[0.010]
15	-0.0177**	-0.0153	-0.0023	-0.0141	-0.0173***	-0.0022	-0.0110	-0.0043	0.0006	-0.0098	-0.0024	0.0007
	[0.009]	[0.010]	[0.009]	[0.009]	[0.006]	[0.007]	[0.007]	[0.005]	[0.010]	[0.007]	[0.007]	[0.010]
Gender & Race				X	X	X	X	X	X	X	X	X
Entry Occupational & Industrial Groups							X	X	X	X	X	X
ASVAB Test Scores										X	X	X
Adjusted R-squared	0.231	0.236	0.234	0.267	0.270	0.268	0.299	0.304	0.299	0.311	0.316	0.312

Clustered standard errors in brackets
* p<0.10, ** p<0.05, *** p<0.01

Table 5: Quantile Regression Results

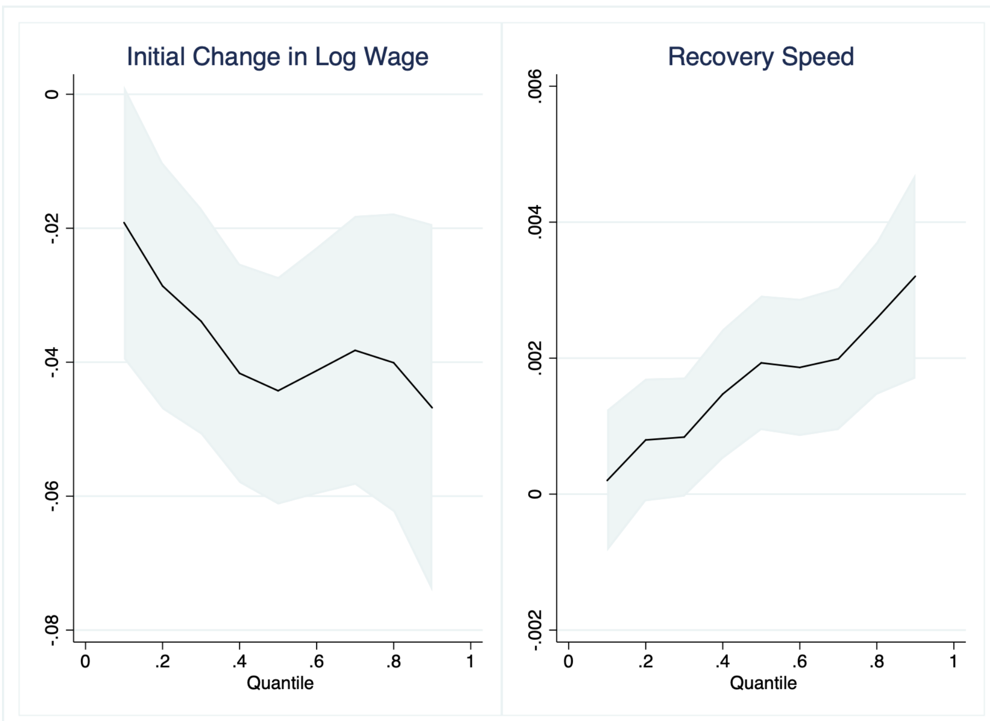


Figure 2: Initial Wage Loss (Left) & Recovery Speed (Right)

speed for workers in all residual wage deciles under the baseline model (see Figure 2). The shaded areas present the 85% confidence intervals for the first to ninth deciles. I find that graduating and entering the labor market during the recession negatively impacts the entry wages of high school graduates. However, there is no significant heterogeneity across workers in different deciles. However, there are significant differences in recovery speed for workers across quantiles. The findings are consistent under cases with additional controls. In the next section, I present a model to explain the reasons for heterogeneity in recovery speed.

4 Theoretical Model

So, how does a short-run aggregate shock (i.e., recession) have long-term effects on young workers' labor market outcomes? To address this question, I develop a general equilibrium model featuring a representative firm consisting of different occupations and heterogeneous workers. Consistent with Lazear (2009), I assume occupations differ in their requirements for multidimensional general skills used in production. In my model, the firm produces the final goods, which require all intermediate goods produced by occupations. These occupations compete for workers by offering contracts based on their skill sets. For the firm, its problem can be simplified to find optimal contract offers to workers in each occupation. Then, workers choose to work in the occupation that offers them the highest wage and decline offers from other occupations.

On the workers' side, they endogenously choose their optimal occupations based on wage offers and decide investments on multidimensional human capital accumulation over their lifetimes, accounting for both aggregate shocks in skills productivity and idiosyncratic learning shocks.[8] In this model, I simulate a recession as the productivity of skills declines in the year of entry, so incorporating short-run aggregate shock allows me to discuss the effects of a recession on workers' remaining lifetime wages. Much like Cavounidis and Lang (2020), I incorporate workers' skill accumulation processes, following the conventional Ben-Porath model, while extending the process with workers' idiosyncratic learning about their true skill-accumulation abilities.

The idiosyncratic shocks come from individuals only having partial information about their true skill accumulation abilities through test scores. Besides, in this model, I also consider risk-averse agents that are heterogeneous in three dimensions: true skill accumulation abilities, initial signals on abilities, and initial human capital.[9] By including the differences in initial signals, it is to account for the possibility that the same-ability workers may receive very biased signals about their true abilities, which lead to different behaviors and life-cycle outcomes due to recession. The details of the model are illustrated below.

4.1 Firm, Occupation, and Technology

I assume a representative firm produces a single final good, with the firm consisting of different occupations. The production of the final good utilizes all intermediate goods produced in occupations. Following Cavounidis and Lang (2020), I consider the final goods production function as a linear combination of all intermediate goods, represented as

$$Y_t = \sum_m x_{m,t}. \tag{2}$$

[8] In this paper, I use human capital and skill interchangeably.

[9] Huggett et al. (2011) discusses the sources of lifetime inequality differences and finds that initial human capital and true abilities are essential in explaining the life-cycle earnings patterns. They also consider wealth heterogeneity in the paper since they are multiple cohorts, which turns out to be a minor factor in explaining the lifetime inequality in earnings.

I denote $x_{m,t}$ as the quantity of the intermediate good produced by an occupation m in the representative firm at time t. I normalize the price of the final good to 1. In principle, I should also specify the prices of each intermediate good, but they are all the same as the price of the final good due to the linear production and competitive final good market assumptions. So, I can even think of $x_{m,t}$ as the quantity of the final goods produced by each occupation m at time t.

Within each occupation, workers need a set of N skills to complete tasks, with each skill indexed as $n \in \{1, 2, ..., N\}$. I assume a continuum of occupations, each characterized by a unique skill intensity vector. For a specific occupation m, the skill intensity vector is denoted as $\boldsymbol{J}_m = (J_{m,1}, J_{m,2}, ..., J_{m,N})$, where the requirement for any skill n is constrained within the range of 0 to 1, inclusively ($0 \leq J_{m,n} \leq 1$). It is important to note that these skill intensity requirements for each occupation remain consistent over time. In other words, an occupation m is characterized as a vector $\boldsymbol{J}_m$ of weights applied to skills.

Following Cavounidis and Lang (2020), the skill intensity vector of any occupation m must satisfy the following technological constraint to be feasible to produce outputs

$$\sum_{n=1}^{N} J_{m,n}^{\sigma} \leq 1, \tag{3}$$

where σ ($\sigma > 1$) determines the sets of skills requirement bundle for the frontier occupations. Frontier occupations, defined by skill intensities for which the equality holds in equation (3), exhibit a unique attribute: they represent the upper bounds of skill requirements, with no other occupation requiring higher intensities for both skills simultaneously. In comparing two frontier occupations, if a particular frontier occupation demands a higher skill intensity for one skill, the intensity requirement for the other must be lower.

The value of σ also reflects the benefit of combining skills in frontier occupations' production. For example, if I only have two skills ($N = 2$) and as σ approaches 1, the feasible occupations satisfying equation (3) are in the lower triangle of the first quadrant in Cartesian coordinates. All frontier occupations lie on the hypotenuse. If σ increases, the feasible occupation set expands, and the requirements for all skills in production by frontier occupations also increase. When $\sigma \to \infty$, the feasible set is the whole square with a side length of one, with frontier occupations requiring both skills in the production at maximum intensity. Therefore, the level of σ signifies the advantages of skills combinations in production. I denote Ω as the set of feasible occupations satisfying the technological constraint (equation (3)).

One could argue that the importance of skills in an occupation changes over time due to technological advancements, which contradicts my assumption in the model. For instance, Cortes et al. (2023) documents how the change in relative task importance, which itself determines occupational sorting based on workers' comparative advantages, has evolved in the U.S. I capture this aspect in this paper by introducing the time-varying skill-specific productivity into the model.

The productivity of different skills reflects how the current technology applies each skill. I denote the skill productivity vector at time t as $\boldsymbol{A}_t = (A_{t,1}, A_{t,2}, ..., A_{t,N}) \in \mathbb{R}_{+}^{N}$. The multiplication of time-varying productivity and occupation-specific skill weight provides insights into how the importance of skills changes over time within an occupation.

The production function for each occupation is linear in the product of skill productivity, occupational skill intensity requirements, and the total skill stock of workers in the occupation. Consequently, production technologies vary across occupations due to their distinct skill intensity vector for utilizing skills in production. The production function in occupation m at time t is

$$x_{m,t} = \sum_{n=1}^{N} A_{t,n} J_{m,n} \bar{H}_{m,t,n}, \tag{4}$$

where $\bar{H}_{m,t,n}$ denotes the total amount of stock of skill n from workers in a specific occupation m at time t. It is important to note that workers cannot sell their skills individually to different occupations. When an occupation decides to hire workers, it inherently utilizes the skills bundle possessed by the workers for production. Thus, a linear production function in each occupation does not imply that skills are perfect substitutes for one another.

In every period, all occupations compete for labor by offering contracts for compensation of skills to workers. I denote $\boldsymbol{R}^*_{m,t} = (R^*_{m,t,1}, R^*_{m,t,2}, ..., R^*_{m,t,N}) \in \mathbb{R}^N_+$ as the compensation of skills offered by occupation m in period t. In my model, individuals supply their skills as a bundle to a specific occupation.

4.2 Household

I assume each worker stays in the labor market for $T+1$ periods, and they are risk-averse. In each period, individuals only value consumption. The utility function is Constant Relative Risk Aversion (CRRA) $u(c) = \frac{c^\eta - 1}{\eta}$, where $1-\eta$ represents the coefficient of constant relative risk-aversion and $\eta < 1$.

Workers are heterogeneous in skill accumulation abilities, initial signals about true abilities of skills accumulation, and initial stock of skills at period 0. For worker i, the skill accumulation abilities vector is denoted as $\boldsymbol{\theta}^i = (\theta^i_1, \theta^i_2, ..., \theta^i_N) \in \mathbb{R}^N_+$ and the stock of skills in period 0 is $\boldsymbol{H}^i_0 = (H^i_{0,1}, H^i_{0,2}, ..., H^i_{0,N}) \in \mathbb{R}^N_+$. I assume the each stock of skill $H^i_{0,n}$ follows a truncated normal distribution with mean μ_{H_n} and variance $\sigma^2_{H_n}$ lies within the interval $[h_{l,n}, h_{u,n}]$.

At time t, worker i goes to the labor market and observes the compensations of skills in all occupations $\{\boldsymbol{R}^*_{m,t}\}_{m \in \Omega}$ (equation (11)). At time t, worker i knows her current human capital bundle to compute the wage rate in each occupation. Her wage rate w^i_t in a occupation m at time t follows

$$w^i_{m,t} = \sum_{n=1}^{N} R^*_{m,t,n} H^i_{t,n}. \tag{5}$$

Each worker chooses the occupation which offers the highest wage rate. After workers decide their optimal occupations, they allocate time to work and invest in skills accumulation.

I assume each person endows one unit of time every period and chooses to allocate between working and skills accumulation. The time spent on accumulation of skills is denoted as $\boldsymbol{q}_t = (q_{t,1}, ..., q_{t,N})$, where $0 \leq q_{t,n} \leq 1 (\forall n = 1, ..., N)$. In this paper, I only focus on the workers' skill investments, so I assume skill investment is the only way to save for the future. Thus, the budget constraint for worker i at time t is

$$c_t^i = (1 - \sum_{n=1}^{N} q_{t,n}) \max_{m \in \Omega} (w_{m,t}^i), \tag{6}$$

where the household has the standard time constraint

$$\begin{aligned} &0 \leq \sum_{n=1}^{N} q_{t,n} \leq 1, \\ &0 \leq q_{t,n} \leq 1, \quad \forall n = 1, ..., N. \end{aligned} \tag{7}$$

With a slight abuse of the notation, I denote worker i's optimal occupational choice at time t as $\boldsymbol{J}_t^i = (J_{t,1}^i, J_{t,2}^i, ..., J_{t,N}^i)$ and time investment on skills as $\boldsymbol{q}_t^i = (q_{t,1}^i, q_{t,2}^i, ..., q_{t,N}^i)$.

The accumulation equation for individual i in each skill n follows standard Ben-Porath setup, except there is an idiosyncratic learning process about true skill accumulation abilities,

$$H_{t+1,n}^i = (1 - \delta) H_{t,n}^i + (\theta_n^i + \epsilon_{t,n}^i)(q_{t,n}^i H_{t,n}^i)^\alpha, \quad \forall n = 1, ..., N. \tag{8}$$

I denote δ as the general depreciation rate of skills, and α $(0 < \alpha < 1)$ determines the degree of diminishing marginal returns in the human capital production function. When α is low, it implies higher diminishing returns, and it is optimal to spread the investment over time. In contrast, when α is high, it is optimal to bunch investment over time. $\epsilon_{t,n}^i$ is the random disturbance term where $\epsilon_{t,n}^i \sim \mathcal{N}(0, \sigma_\epsilon^2)$. Its role in the Bayesian updating process is illustrated in section 4.2.1. Intuitively, the disturbance term captures that the realization of future stock of skills is still affected by some idiosyncratic randomness on learning (e.g., health condition). So, individuals can not backward engineering the true skill accumulation abilities even if they know the current and future stock of skills and the current time investment on each skill. To simplify the notations, I denote $\tilde{\boldsymbol{\theta}}_t^i = (\tilde{\theta}_{t,1}^i, \tilde{\theta}_{t,2}^i, ..., \tilde{\theta}_{t,N}^i)$ where $\tilde{\theta}_{t,n}^i = \theta_n^i + \epsilon_{t,n}^i$ for each skill n, and I use this notation in the rest of the paper.

4.2.1 Information Setup

Aggregate Shocks on Productivity of Skills. Households do not have full information about their abilities and the true productive efficiency of skills. In this environment, I assume households take standard expectation on the shocks, and the productivity of independent skills follow the log-AR(1) process

$$\log(A_{t+1,n}) = (1 - \rho)\mu_{A_n} + \rho \log(A_{t,n}) + \gamma_{t,n}, \quad \forall n = 1, ..., N \tag{9}$$

where $\gamma_{t,n} \sim \mathcal{N}(0, \sigma_\gamma^2)$ and ρ is the persistence level.

Idiosyncratic Learning Processes. When worker i is born, her draws skills accumulation ability set $\boldsymbol{\theta}^i$, and ability of accumulating each skill n follows a normal distribution: $\theta_n^i \sim$

$\mathcal{N}(\mu_{\theta_n}, \sigma_\theta^2)$. Before the worker entering the labor market, the worker i does not observe the true ability set $\boldsymbol{\theta}^i$ but observe signals $\hat{\boldsymbol{\theta}}_0^i = (\hat{\theta}_{0,1}^i, \hat{\theta}_{0,2}^i, ..., \hat{\theta}_{0,N}^i)$. Based on the signals for all skills, this individual forms a prior belief about the distribution of each skill $\theta_n^i = \hat{\theta}_{0,n}^i + \tau_n^i$, where $\tau_n^i \sim \mathcal{N}(0, \sigma_{\tau_n}^2)$. I define the initial precision as $\lambda_{0,n} = \frac{1}{\sigma_{\tau_n}^2}$.

Then, at each period t, workers make dynamic choices on time investment on skills accumulation by using the prior belief about their skill accumulation abilities. Once they finalize their time investment choices, I assume worker i observes the realization of $\tilde{\boldsymbol{\theta}}_t^i$. I define the precision as $\lambda_\epsilon = \frac{1}{\sigma_\epsilon^2}$. Based on the observation, the worker i updates the belief about $\boldsymbol{\theta}^i$ and uses the posterior belief as the prior belief in the next period. The updating law of motion for each skill n follows

$$\hat{\theta}_{t+1,n}^i = \frac{\lambda_{t,n}}{\lambda_{t+1,n}} \hat{\theta}_{t,n}^i + \frac{\lambda_\epsilon}{\lambda_{t+1,n}} \tilde{\theta}_{t,n}^i, \quad \forall n = 1, ..., N \tag{10}$$

where $\lambda_{t+1,n} = \lambda_{t,n} + \lambda_\epsilon$ with $\lambda_{0,n} = \frac{1}{\sigma_{\tau_n}^2}$ and $\lambda_\epsilon = \frac{1}{\sigma_{\epsilon_n}^2}$.

Proposition 1 *For any skill n, the precision $\lambda_{t,n}$ increases and $Var(\hat{\theta}_{t,n}^i - \theta_n^i)$ declines over time.*

Proof. See Appendix A.4.1. □

Thus, workers' prior beliefs about their abilities become more precise over time. It implies that more experienced or older workers have better self-awareness of their true skill accumulation abilities.

4.3 Timing Within A Period

The sequence of events within each period t is illustrated in Figure 3. For illustrative purposes, I can think of a period as divided into three subperiods. In this figure, I use a randomly chosen worker i as an example for illustration purposes, and it applies to all workers.

- *Subperiod 1.* Given the current productivity of skills $\boldsymbol{A_t}$, and the stock of skills of any worker i ($\boldsymbol{H}_t^i$), occupations are competing workers by offering wage contracts, $\{\boldsymbol{R}_{m,t}^*\}_{m\in\Omega}$.

- *Subperiod 2.* For each worker i, she observes the current stock of skills $\boldsymbol{H}_t^i$ and the contract offered by all occupations $\{\boldsymbol{R}_{m,t}^*\}_{m\in\Omega}$. She can determine the optimal occupational choice $\boldsymbol{J}_t^i$ by maximizing the offered wage rate. For the dynamic choices on time investment for human capital accumulation, worker i also forms prior beliefs about the true skill accumulation abilities $\hat{\boldsymbol{\theta}}_t^i$. However, worker i has no information either about the future productivity of skills $\boldsymbol{A}_{t+1}$ or true realization of idiosyncratic shocks $\tilde{\boldsymbol{\theta}}_t^i$. Thus, she chooses optimal time investment on skills accumulation $\boldsymbol{q}_t^i$ and consumption c_t^i by taking expectation on $\boldsymbol{A}_{t+1}$ and true skill accumulation abilities based on prior beliefs.

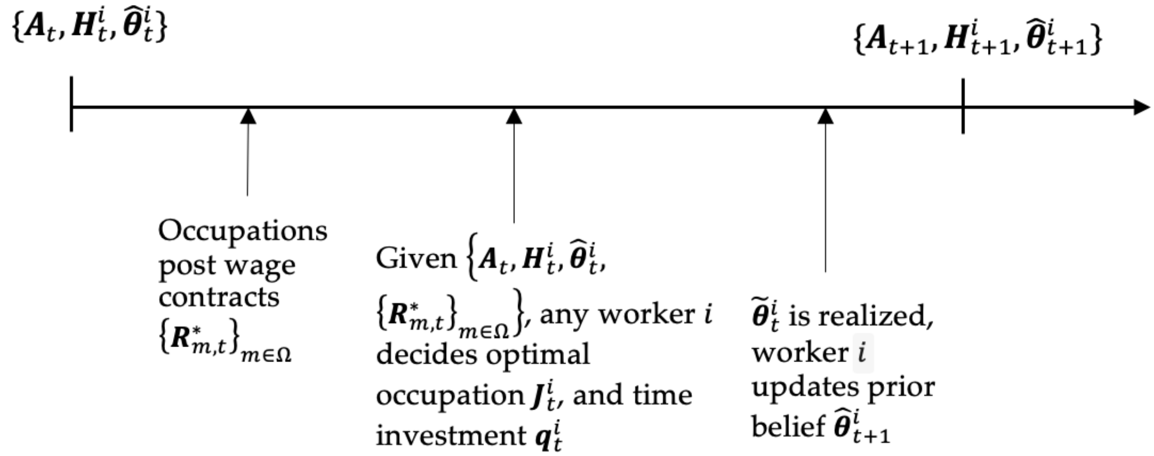


Figure 3: Timing Within One Period

- *Subperiod 3.* For each worker i, after determining the optimal choices, both shocks on skills' productivity $\boldsymbol{A}_{t+1}$ and idiosyncratic learning process $\tilde{\boldsymbol{\theta}}_t^i$ are realized. Worker i updates her beliefs about true skill accumulation abilities and form the posterior belief $\hat{\boldsymbol{\theta}}_{t+1}^i$ based on the equation (10). In the following period $t+1$, worker i uses these posterior beliefs as new prior beliefs about true abilities.

4.4 Occupation's Problem

Suppose a worker i with a certain set of skill levels enters the job market at time t. Given her stock of skills, she is looking for an occupation that offers her the best wage among all offers. Each occupation, denoted as m, presents a contract $\boldsymbol{R}^*_{m,t}$ in period t aimed at maximizing its profit given the set of skills of worker i.

$$\max_{\boldsymbol{R}^*_{m,t}} \left[\sum_{n=1}^{N} A_{t,n} J_{m,n} H_{t,n}^i - \sum_{n=1}^{N} R^*_{m,t,n} H_{t,n}^i \right]$$

Proposition 2 *Workers only work in the frontier occupations.*

Proof. See Appendix A.4.2. □

This result is driven by the labor market structure and the assumption of a continuum of occupations. For any non-frontier occupation, there is always a frontier occupation that provides workers with better contracts and attracts them.

Proposition 3 *The optimal contract in occupation any $m \in \Omega$ is $\boldsymbol{R}^*_{m,t} = (R^*_{m,t,1}, R^*_{m,t,2}, ..., R^*_{m,t,N})$ where*

$$R^*_{m,t,n} = A_{t,n} J_{m,n}, \qquad \forall n = 1, ..., N \tag{11}$$

Proof. See Appendix A.4.3. □

Thus, only frontier occupations are competitive in hiring labor since they can offer better contracts, and the compensation for each skill depends only on aggregate productivity and occupation-specific skill requirements. In optimum, all occupations gain zero profit due to the competition.

4.5 Worker's Problem

Worker i's problem: Given the compensations of skills menu $\{\boldsymbol{R}^*_{m,t}\}_{m\in\Omega}$ offered by all feasible occupations, the household solves the following problem in period t:

$$V_t(\boldsymbol{A}_t, \boldsymbol{H}^i_t, \hat{\boldsymbol{\theta}}^i_t) = \max_{\{\boldsymbol{q}_t, \boldsymbol{J}_t\}} \left\{ \frac{(c^i_t)^\eta - 1}{\eta} + \beta \mathbb{E}_t V_{t+1}(\boldsymbol{A}_{t+1}, \boldsymbol{H}^i_{t+1}, \hat{\boldsymbol{\theta}}^i_{t+1}) \right\}$$

$$s.t. \quad \text{Eq. (11)} - \text{Eq. (10)}$$

$$\boldsymbol{A}_t, \boldsymbol{H}^i_t, \hat{\boldsymbol{\theta}}^i_t \text{ are given,}$$

where V_t is the value function in period t. The current state variables are $\{\boldsymbol{A}_t, \boldsymbol{H}^i_t, \hat{\boldsymbol{\theta}}^i_t\}$ and choice variables are $\{\boldsymbol{q}^i_t, \boldsymbol{J}^i_t\}$.

Since the occupational choice is static, there exists a close-form solution. As for the time investment in skills, there is a trade-off between consumption today and tomorrow. Holding other things constant, if the time investment in one skill increases, the cost is the decline in current consumption, but the benefit is higher skill levels, which leads to higher future consumption. Since the human capital accumulation follows the Ben-Porath setup, a change in the time investment in skills affects both sides of the Euler equation. Moreover, in this multidimensional skill model, the marginal cost of investment in each skill is the same. Since there is no closed-form solution, I solve the problem numerically from backward.

Proposition 4 *The optimal occupational choice for worker i in period t is* $\boldsymbol{J}^i_t = (J^i_{t,1}, ..., J^i_{t,N})$, *where*

$$J^i_{t,n} = \frac{(A_{t,n} H^i_{t,n})^{\frac{1}{\sigma-1}}}{\left[\sum_{n=1}^N (A_{t,n} H^i_{t,n})^{\frac{\sigma}{\sigma-1}}\right]^{\frac{1}{\sigma}}}, \quad \forall n = 1, ..., N \tag{12}$$

and the optimal wage rate for worker i in period t is

$$w^i_t = \left[\sum_{n=1}^N (A_{t,n} H^i_{t,n})^{\frac{\sigma}{\sigma-1}}\right]^{\frac{\sigma-1}{\sigma}}. \tag{13}$$

Proof. See Appendix A.4.4. □

Given the productivity of skills, $\frac{\partial J^i_{t,s}}{\partial H^i_{t,s}} > 0$ and $\frac{\partial J^i_{t,s^-}}{\partial H^i_{t,s}} < 0$. Thus, compare worker i and j, if $H^i_{t,s} > H^j_{t,s}$ but $H^i_{t,s^-} = H^j_{t,s^-}$, worker i tends to work in occupation with higher intensity of skill s. This finding is consistent with Cavounidis and Lang (2020)'s paper.

Given the stock of skills, I have $\frac{\partial J^i_{t,s}}{\partial A_{t,s}} > 0$ and $\frac{\partial J^i_{t,s^-}}{\partial A_{t,s}} < 0$, which imply workers tend to follow the trend of productivity when they choose their optimal occupations. Holding other things constant, workers choose occupations with less intensity of skill s but higher intensity of other skills if the productivity of skill s decreases. Moreover, suppose the productivity of skills has the same percentage decline in a recession. The workers still have the same occupation in the year of recession but have different occupational paths and wages afterward due to the changes in the stock of skills.

4.6 Equilibrium

In this model, there are markets for intermediate goods, labor, and the final good. The market-clearing prices of all intermediate goods are one due to the linear production function assumption of the final good. Because of the timing assumption regarding occupations' decisions and the labor market structure, the general equilibrium outcomes in the model align with the optimal choices of workers in partial equilibrium as if they observe optimal contracts offered by all occupations that satisfy Proposition 3 (see Cavounidis and Lang (2020)). In other words, in the labor market equilibrium, occupations determine prices, and then workers assign themselves to occupations and work based on their comparative advantages on the skills bundle. The final good market is perfectly competitive, and the final good market clears due to Walras's law.

4.7 Model Implications

Corollary 1 *Suppose worker i and j work in the same occupation at time t, and their stock of skill difference is a constant ratio across different skills $\frac{H^i_{t,n}}{H^j_{t,n}} = \rho_{ij} (\forall n = 1, ..., N)$; their wage rates difference is also the same constant ratio $\frac{w^i_t}{w^j_t} = \rho_{ij}$.*

The model predicts that workers in the same occupation have parallel stocks of skill vector $\boldsymbol{H_t}$, and the scalar can be computed from the wage ratio from different workers in any period t. The prediction is similar to the idea in Roy (1951), which states that workers in the same occupation have the same comparative advantage over skills. Since I introduce productivity of skill-specific technology, I define comparative-advantage skills as the value of skills, the multiplication of productivity and skill level, not the levels of skills only. The wage differences within occupations are due to the absolute value differences in the stock of skills among different workers.

Corollary 2 *The simulated wage loss due to recession for worker i at time t is*

$$\Delta \ln(w^i_t) = \ln(w^{i,R}_t) - \ln(w^{i,NR}_t) = \frac{\sigma - 1}{\sigma} \ln \left[\frac{\sum_{n=1}^{N} (A^R_{t,n} H^{i,R}_{t,n})^{\frac{\sigma}{\sigma-1}}}{\sum_{n=1}^{N} (A^{NR}_{t,n} H^{i,NR}_{t,n})^{\frac{\sigma}{\sigma-1}}} \right],$$

where R, NR stands for recession and non-recession separately.

Additionally, since individual optimal wage rates in each period only relate to two components, the model is plausible to discuss the two main reasons leading to the scarring effect on young workers on their life-cycle wages: First, the aggregate productivity of skills declines due to recession even if workers do not change their human capital investments. Second, because of the decline in skills productivity, workers decide to increase or decrease their time investments in skills, leading them to different occupations and wages. In the following section, I will conduct a counterfactual experiment to separate these two reasons. Moreover, if we compare heterogeneous workers, the differences in their recovery speed are only related to their initial endowments on these three dimensions related to skill accumulations. In the following section, I also discuss the critical factor that leads to the heterogeneity in recovery speed.

5 Quantitative Analysis

I now describe the numerical analyses from my calibrated model. With these numerical solutions, I elucidate the primary factors responsible for the scarring effect on young workers' wages over the life cycle. I quantitatively determine the extent to which changes in skill productivity and variations in workers' investments in skill accumulation contribute to this effect. Additionally, I explore the main factor that leads to unequal recovery speed among workers.

In the following section, I first outline the calibration strategy utilized in this paper and discuss the model's fit. Then, I conduct simulations to replicate an economic scenario similar to the early 1980s recession and compute the long-term scarring of workers' wage patterns over the life cycle. I describe the numerical results from a calibrated model and numerically determine the role of signals by comparing results to an economy where workers have precise signals about their true abilities. Additionally, I decompose the contribution of the two reasons leading to scarring effects by conducting a counterfactual experiment. Lastly, I present the lifetime wage losses by heterogeneous workers due to the recession and discuss the main factor that leads to disparities in recovery speed.

5.1 Calibration

To calibrate the model to the data, I require additional information on occupational skill requirements and workers' initial signals about their true abilities. For simplicity, I focus on two skills: measurable intellectual skill (Skill 1) and unmeasurable skill (Skill 2). I obtain information on occupational skill requirements from the Occupational Information Network (O*NET) data set for this paper. For workers' initial signals about their true abilities, I acquire this information from their ASVAB test scores. However, a challenge arises because these two data sets use different skill measurements. I provide a detailed explanation of the O*NET data set and the procedure for linking ASVAB skills with O*NET measurements in Appendix A.3.

I categorize the parameters into two groups. The first group includes parameters that can be directly determined using data, while the second group consists of calibrated parameters obtained through the simulated method of moments. I refer to the parameters in the first group as fixed parameters, while the ones in the second group are called free parameters.

PARAMETER	DEFINITION	VALUE	TARGET
T	Time length	49	Average years of working
η	1-relative risk aversion	-1.00	from Huggett et al. (2011)
β	Discount factor	0.98	$\beta = 1/(1+r)$
α	Degree of diminishing marginal return in skill accumulation (elasticity parameter)	0.55	$\alpha \in (0.5, 0.9)$ from Browning et al. (1999)
σ	Utilization of skills in frontier occupations	3.20	Frontier occupations
$\{\mu_{\theta_n}\}_{n=1}^2$	Mean of true accumulation abilities of 2 skills	$\{1.87, 1.31\}$	Initial distributions of workers' transformed test scores in 2 skills
$\{A_{0,n}\}_{n=1}^2$	Initial productivity of 2 skills	$\{e^{\mu_{A_1}} e^{\mu_{A_2}}\}$	Assumption
μ_{A_2}	Mean of log productivity of skill 2.	0	Normalization
$h_{l,2}$	Lower bound of the truncated normal distribution for initial stock unmeasurable skill	3.72	Initial distribution for the values of unmeasurable skill $(A_{0,2}H^i_{0,2})$: Minimum; Maximum; Average; Standard Deviation
$h_{u,2}$	Upper bound of the truncated normal distribution for initial stock of unmeasurable skill	16.07	
μ_{H_2}	Mean of the truncated normal distribution for initial stock of unmeasurable skill	7.21	
σ_{H_2}	Standard deviations of the truncated normal distribution for initial stock of unmeasurable skill	2.69	

Table 6: Fixed Parameters

Group 1: Fixed Parameters. This paper focuses on the life-cycle behaviors of workers in the labor market. I assume that workers enter the labor market at the age of 18, which leads me to set $T = 49$ to align with the average retirement age of 67 for individuals born after the 1960s in the United States. [10]

Regarding preference-related parameters, I set the annual net interest rate r at 2% and derive the inter-temporal discount factor as $\beta = \frac{1}{1+r} = 0.98$. Additionally, I adopt a relative risk aversion of 2, following Huggett et al. (2011), setting $\eta = -1$. To account for diminishing marginal returns in skill accumulation, I assign $\alpha = 0.55$, a value within the reasonable range of 0.5 to 0.9, as reported in Browning et al. (1999). For σ, I use an estimate of 3.20, which closely matches how frontier occupations utilize different skills in production (further details are provided in Section A.3.1).

In the model, I define the initial prior beliefs about skill accumulation ability n of every individual as following a normal distribution: $\hat{\theta}^i_{0,n} = \theta^i_n - \tau^i_n \sim \mathcal{N}(\mu_{\theta_n}, \sigma^2_{\theta_n} + \sigma^2_{\tau_n})$. I determine the means to be $\mu_{\theta_1} = 1.87, \mu_{\theta_2} = 1.31$ based on the initial distributions of

[10] These statistics are sourced from the *Retirement Plans* document published by the Social Security Administration in 2022, which outlines retirement ages by birth year. You can access the document here `https://www.ssa.gov/pubs/EN-05-10035.pdf`.

workers' test scores in two skills. In this study, I assume the economy starts with the mean productivity of skills, resulting in $\{A_{0,n}\}_{n=1}^{2} = \{e^{\mu_{A_1}}, e^{\mu_{A_2}}\}$. I normalize the mean log productivity of the unmeasurable skill ($n = 2$) to equal 0.

Based on the optimal compensation of skills equation (11), wage equation (13), and the optimal occupation and wage from proposition (12), the value of any skill n for worker i ($A_{t,n}H^i_{t,n}$) at period t can be computed as follows:

$$\ln(A_{t,n}H^i_{t,n}) = \ln(w^i_t) + (\sigma - 1)\ln(J^i_{t,n}).$$

I obtain the hourly wage rate and the occupational skill requirements for each individual at time t, and with $\delta = 3.20$, I can calculate the value of each skill for every worker-year observation. To calibrate parameters related to the truncated normal distribution of the initial stock of unmeasurable skill, I determine the minimum, maximum, and two moments from the initial distribution for the values of that skill, $A_{0,2}H^i_{0,2}$.

In principle, I can measure $h_{l,n}$ from the minimum values of skill n ($h_{l,n} = \frac{\min(A_{0,n}H^i_{0,n})}{A_{0,n}}$) and $h_{u,n}$ from the maximum values of skill n in the data ($h_{u,n} = \frac{\max(A_{0,n}H^i_{0,n})}{A_{0,n}}$). The comparison between simulated and the actual initial distribution of the value of unmeasurable skill is shown in Figure 4.[11] Since $A_{0,1}$ is currently unknown, I can only externally calibrate the distribution of the unmeasurable skill. However, the method is the same for both skills.

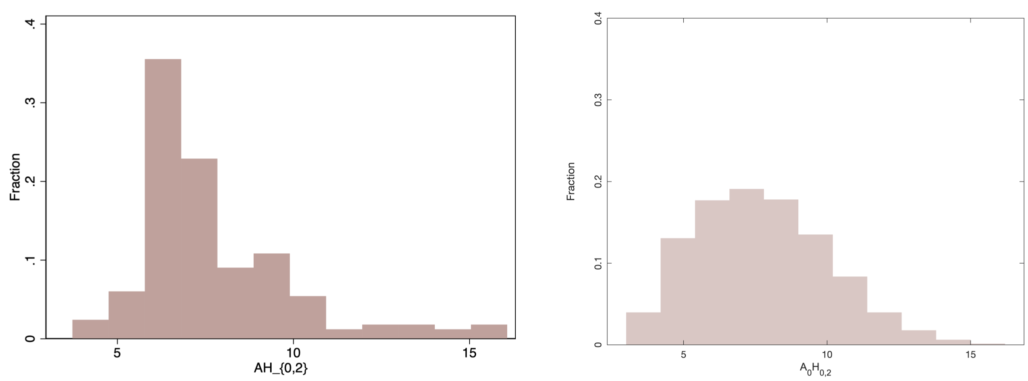


Figure 4: Distribution of Unmeasurable Skill Value: Data (Left) & Simulation (Right)

Group 2: Free Parameters. In this section, I jointly calibrate eleven parameters listed in Table 7 using the simulated method of moments. To compute all moments in the model, I simulate life-cycle sequences of random shocks on learning and skill productivity processes 500 times.

First, if I obtain the estimate for the mean log productivity of intellectual skill, I can determine all four parameters related to the truncated normal distribution for the initial stock of that skill. This process mirrors what I do for the unmeasurable skill, and, as such,

[11] I conduct 10,000 simulations here.

PARAMETER	DEFINITION	VALUE	TARGET
μ_{A_1}	Mean of log productivity of intellectual skill	-1.80	Relative ratio between skill values over the life cycle
δ	Depreciation of skills	0.22	Curvature of skills values over life cycle
σ_ϵ	Standard deviations of the idiosyncratic shocks in the realization of learning 2 skills	1.60	Speed of decline in gross mobility
σ_γ	Standard deviations of log-AR(1) processes of 2 skills	0.07	Relative gross mobility between young and old
ρ	Persistent level of productivity	0.50	Average scarring effect duration
σ_θ	Standard deviations of true abilities of accumulating 2 skills	0.30	Simulated variance of $\hat{\boldsymbol{\theta}}_{\boldsymbol{T}}$
$h_{l,1}$	Lower bound of the truncated normal distribution for initial stock of intellectual skill	0.06	Initial distribution for the values of intellectual skill $(A_{0,1}H^i_{0,1})$: Minimum; Maximum; Average; Standard Deviation
$h_{u,1}$	Upper bound of the truncated normal distribution for initial stock of intellectual skill	20.51	
μ_{H_1}	Mean of the truncated normal distribution for initial stock of intellectual skill	4.68	
σ_{H_1}	Standard deviations of the truncated normal distribution for initial stock of intellectual skill	5.35	
d	The radius to measure mobility	0.23	Absolute gross mobility

Table 7: Free Parameters

these parameters become functions of μ_{A_1}. The question now is how I estimate μ_{A_1} from the data.

In the data, I can back up the average values of two skills across workers over the life cycle ($A_{t,n}\bar{H}_{t,n}$). The skill depreciation rate δ relates to the curvature of skills values over the life cycle. is connected to the curvature of skill values over the life cycle. Given that I have normalized the mean of log productivity of the unmeasurable skill (μ_{A_2}) to 0, μ_{A_1} will determine the relative value of skills over the life cycle.

To obtain estimators for the other parameters, I rely on the computed probability of switching occupations by age from NLSY79, which I refer to as gross occupational mobility. It takes the form of a downward-sloping curve (for further computation details, see Appendix A.1), which means individuals are more likely to switch occupations when they are young than old. The model suggests that idiosyncratic uncertainty (σ_ϵ) and standard deviations for the initial endowments of skill accumulation abilities (σ_θ) are linked to a worker's Bayesian learning about their true accumulation ability in two skills. The speed of learning is associated with changes in precision over time. This implies that the rate of decline in the gross mobility curve reflects the variation in idiosyncratic learning shocks (σ_ϵ). According to the model, workers gradually come to understand their true skill accumulation abilities as they age. This implication of Bayesian learning helps me determine the distribution of true skill accumulation abilities (σ_θ).

From the model, workers primarily switch occupations due to aggregate uncertainty when they get older. Consequently, I calibrate the standard deviations of the log-AR(1) processes of the two skills (σ_γ) by matching the gross occupational mobility in old age.

In the model simulation, I define an occupational switch as the Euclidean distance of changes in occupational skill intensities that cannot be constrained within a circle with a radius $d(d > 0)$. The choice of a radius d greater than zero is essential because the gross occupational mobility is not 100%. Additionally, with a radius lower than 1, the probability of switching occupations is greater than zero. I obtain $d = 0.23$ by matching the computed gross occupational mobility data across the life cycle.

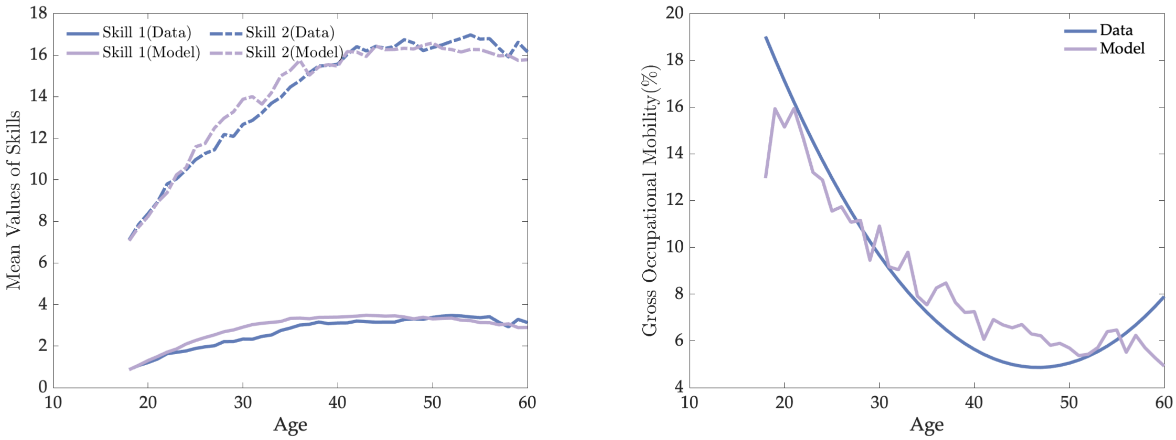
In an economy with higher persistence in skill productivity, it takes longer to recover from a recession, resulting in a more prolonged scarring effect. Therefore, I utilize the average scarring effect duration to calibrate the persistence level $\rho = 0.5$.

Model Fit. The calibration strategy primarily focused on two life-cycle variables: the values of skills and gross occupational mobility. Table 8 lists statistics over the ages of these two variables from data and model simulation. In general, the simulated results align well with the observed average probability of changing occupations over age, and the simulation also captures the similar volatility in gross occupational mobility as in the data. The simulation matches the data for the average skill values and their volatility across different ages in general, except the model generates higher values of measurable intellectual skill.

Statistics	**Data**	**Model**	**Model/Data**
Average Gross Mobility	8.53	8.58	1.01
Relative Stdv of Gross Mobility	0.48	0.37	0.78
Average Values of Skill 1	2.70	2.88	1.07
Average Values of Skill 2	14.12	14.22	1.01
Relative Stdv of of Skill 1	0.27	0.24	0.89
Relative Stdv of of Skill 2	0.21	0.19	0.94

Table 8: Model Fit

To gain deeper insights into the model's accuracy, I present the average values of various variables across different ages throughout the life cycle in Figure 5. Panel (a) illustrates the mean skill value across ages for both measurable intellectual and unmeasurable skills. The solid lines represent the intellectual skill, while the dashed lines represent the unmeasurable skill. The blue lines depict the data patterns, and the mauve purple lines show the simulated patterns derived from the model. Both the data and simulation start at the same point for the mean value of each skill, indicating that the calibrated parameters regarding the distribution of the initial stock of skills align with the initial value of skills. The distributions of the intellectual skill value in both the data and model are further explored in Figure 6, showcasing a comparable resemblance. The relative ratio between the values of the two skills closely matches the data, suggesting an accurate determination of the estimated mean log productivity of the intellectual skill. The curvature observed in the skill value pattern within the simulation closely resembles that in the



(a) Mean of Values of Skills (b) Gross Occupational Mobility

Figure 5: Targeted Life-cycle Variables

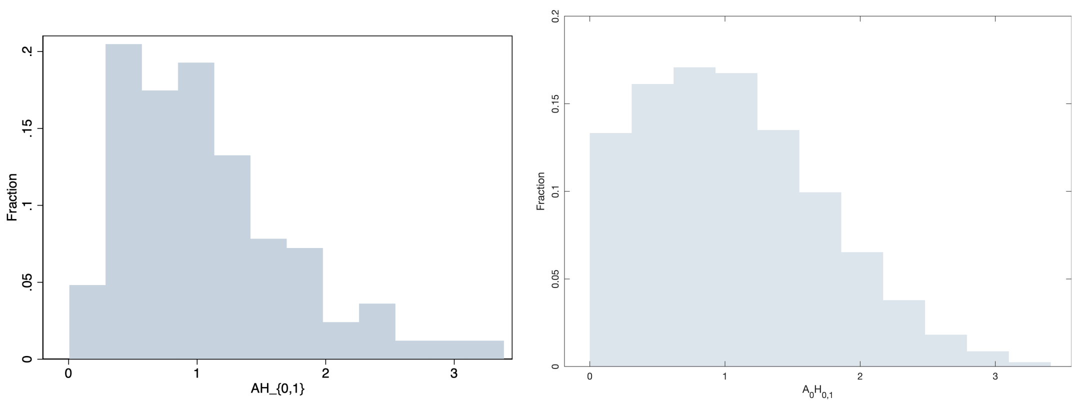


Figure 6: Distribution of Intellectual Skill Value: Data (Left) & Simulation (Right)

data, indicating the calibration of the depreciation rate to match the data. Consequently, the model broadly fits the observed pattern of mean skill values over the life cycle.

Panel (b) in Figure 5 presents the life-cycle gross mobility pattern. The data and simulation are mainly overlapped, except when ages after 50. This divergence arises because I utilize a quadratic prediction for the actual probability of switching occupation trends. Specifically, the actual gross occupational mobility remains approximately 6% after the age of 50. Additionally, I conduct a simulation resembling a recession akin to the early 1980s recession. I plot the wage losses over ages, ultimately determining that the duration of the scarring effect is approximately 15 years, a finding consistent with my empirical observation. I will present this plot in the subsequent section.

To check the validation of the model calibration, I compare the data and simulation results for another untargeted life-cycle pattern: the age-wage profiles. The model also provides reasonable simulation for the life-cycle hourly wage rates (see Figure 7). I plot the quadratic prediction of hourly wage on ages from data in the blue line. The simulated life-cycle hourly wage profile is presented in the mauve purple line. Surprisingly, my

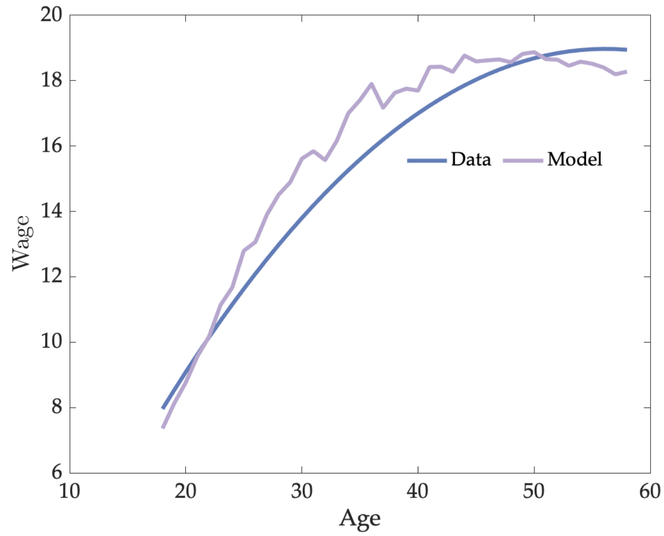


Figure 7: Untargeted Life-Cycle Moments: Real Hourly Wage

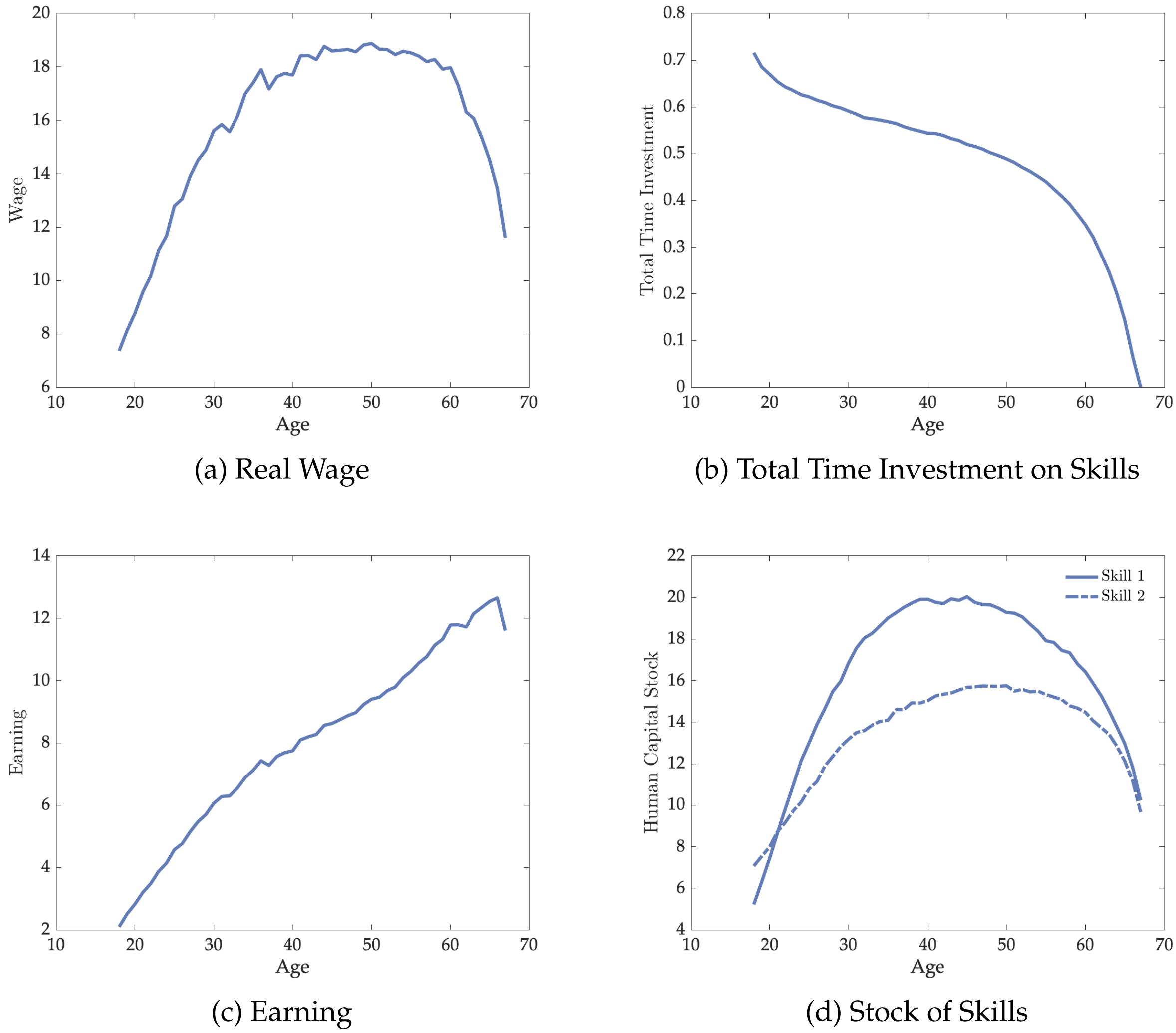
model simulation's curvature is slightly higher, but the overall wage pattern looks similar to the data.

5.2 Mechanics of the Model

Before jumping into the main findings, I illustrate some relevant simulated average life-cycle patterns during what I define as "normal times," representing an economy without recessions or expansions. It is crucial to grasp the typical life-cycle patterns, which are the foundation for comparisons with the recessionary scenarios.

According to Figure 8 panel (a) and (b), I obtain a concave hump-shaped average real wage pattern across workers and a downward curve for average total time investments in skill accumulation across workers. These findings align with the outcomes typically seen in the standard Ben-Porath model with only one skill. In this paper, I compute the earnings as the multiplication of the wage rate and working hours and get a rising life-cycle profile on earnings shown in panel (c). Figure 8 panel (d) presents the average stock of skills across workers. The solid line is for intellectual skill, and the dashed line is for unmeasurable skill. From the calibration results, workers initially have higher endowments in the unmeasurable skill on average, leading to slower growth in the unmeasurable skill stock. Besides, workers have higher abilities to accumulate intellectual skill on average, which is another reason for the higher growth in the intellectual skill stock.

In the model, workers differ in three initial conditions: initial signals about true skill accumulation abilities vector ($\hat{\boldsymbol{\theta}}_{\mathbf{0}}$), initial human capital vector ($\boldsymbol{H}_{\mathbf{0}}$), and true skill accumulation ability vector ($\boldsymbol{\theta}$). Different workers have distinct life-cycle wage profiles and time investment decisions. In the simulation, I discretize each skill in each dimension into three states labeled $\{L, M, H\}$, representing the lowest, middle, and highest, respectively. So, I have nine combinations of states in each dimension. For example, LM represents the workers with the lowest value on skill 1 and the middle value on skill 2 in the particular



(a) Real Wage

(b) Total Time Investment on Skills

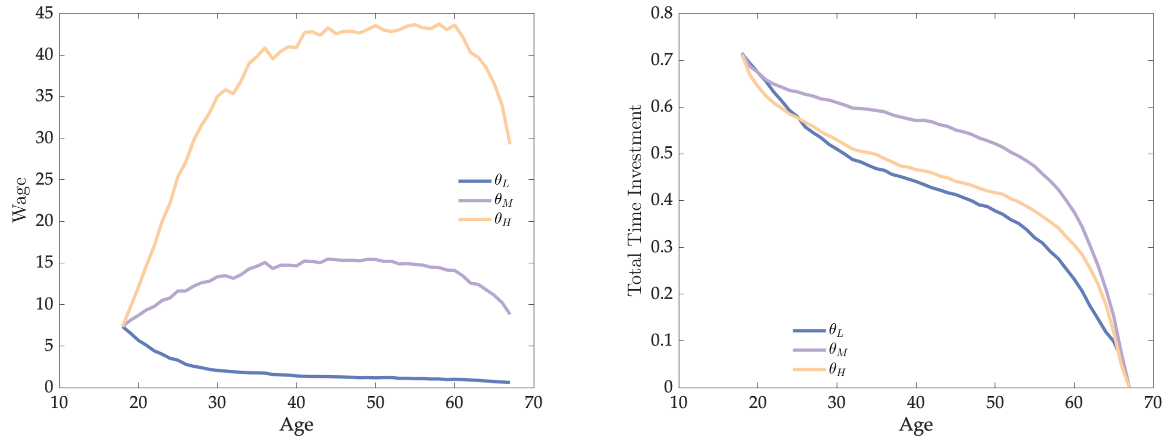
(c) Earning

(d) Stock of Skills

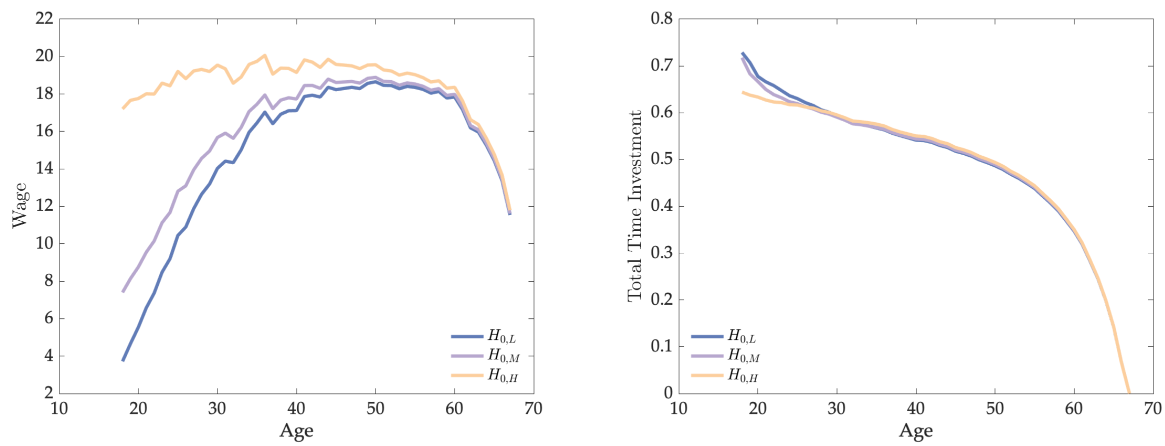
Figure 8: Simulated Life-Cycle Patterns

dimension. Figure 9 presents the life-cycle wage and total time investment on skills accumulation profiles for selected groups of workers. I only selected workers with the lowest, middle, and highest levels in each initial endowment dimension on both skills. For instance, $\boldsymbol{\theta}_L, \boldsymbol{\theta}_M, \boldsymbol{\theta}_H$ indicates workers with the lowest, middle, and highest true abilities on both skills, respectively. Similarly, $\boldsymbol{H}_{0,L}, \boldsymbol{H}_{0,M}, \boldsymbol{H}_{0,H}$ means the workers with the lowest, middle, and highest initial stocks of both skills. $\hat{\boldsymbol{\theta}}_{0,L}, \hat{\boldsymbol{\theta}}_{0,M}, \hat{\boldsymbol{\theta}}_{0,H}$ means the workers with the lowest, middle, and the highest initial signals on both skills. For each panel in Figure 9, I use blue, purple, and yellow lines to represent workers with the lowest, middle, and highest endowments in the corresponding dimension.

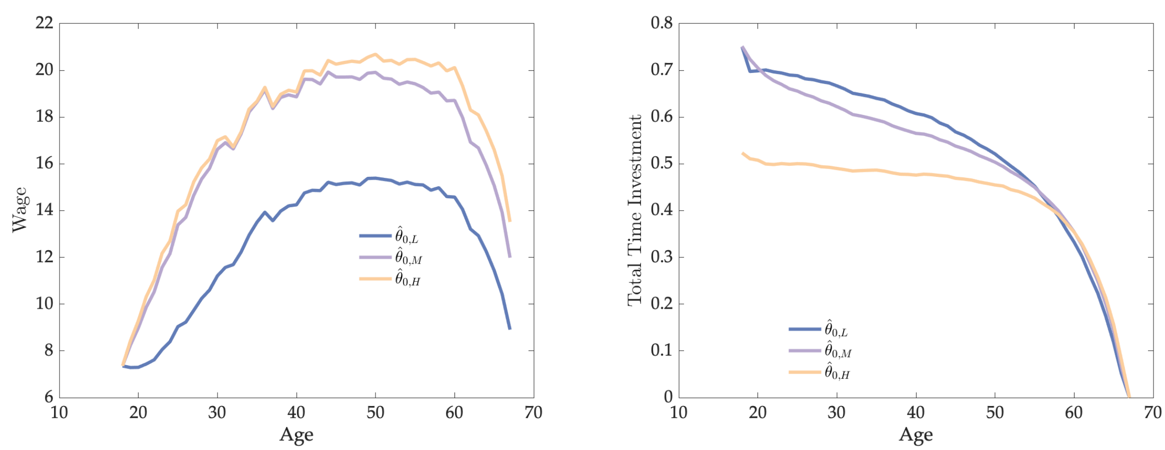
Panel (a) in Figure 9 demonstrates the average wage and total time investments on skills accumulation across workers with different true abilities over the other two dimensions. Workers with all true ability bundles start with the same hourly wage since the average initial stocks of skills are the same. However, the hourly wage profile for workers with the lowest true ability bundles declines over the life cycle. In contrast, the hourly



(a) Average Across True Ability



(b) Average Across Initial Stock of Skills



(c) Average Across Initial Signals

Figure 9: Real Wages (Left) & Total Time Investment (Right)

wage profiles for the middle and highest true abilities groups of workers are inverted-U shape. The wage growth is higher for workers with the highest true abilities bundle. As for the total time investment, workers with the lowest abilities invest less time on skill accumulation than the other two groups of workers over the life cycle. Panel (b) in Figure 9 displays the average hourly wage and total time investments in skill accumulation across workers with different initial skill bundles. While the initial average hourly wages notably differ among workers, these disparities in average wages and total time investments diminish as workers age. Panel (c) in Figure 9 illustrates the differences in average wage and total time investment between workers with varying initial signals. Workers receiving the lowest signals of their true abilities exhibit hourly wage patterns below those receiving higher signals, indicating the costs associated with having low confidence in true skill accumulation abilities. Despite investing more total time in skills accumulation, individuals with the lowest signals still experience lower wage profiles due to changes in their investment portfolio across both skills. These findings align with those in Huggett et al. (2011), highlighting the importance of initial human capital and true abilities in explaining differences in workers' life-cycle patterns.

5.3 Recession Simulation and Scarring Effect

From section 3, I estimate the initial wage loss is around 4% for one percentage point increases in the unemployment rate for those high school graduates who graduated and entered the labor market during the early 1980s recession. From the data, the unemployment rate increased by 2.5 percentage points in the early 1980s recession, leading to a 10% decline in the initial hourly wage. In this paper, I simulate the recession as the productivity of both skills declines by 10% compared to normal times and then recovers to normal following the Log-AR(1) process. The recession occurs when workers enter the labor market $t = 0$. Figure 10 shows the simulated percentage changes in the productivity of two skills during a recession in comparison to the non-recessionary scenario. In both plots, the blue line labeled "*NL*" represents the case when the economy is in the non-recessionary period of time. Conversely, the light purple line labeled "*REC*" corresponds to the situation when the economy is impacted by a recession starting in year 0. I assume that the productivities of both skills follow a Log-AR(1) process with the same persistence level and variance of shocks, resulting in the recovery of both skills' productivities to non-recessionary levels by year 7.

Figure 11 plots the average values (panels (a,b)) and average percentage changes (panels (c,d)) in hourly wage and total time investment throughout the lifetime across workers. In all panels, the labels "*NL*" and "*REC*" have the same meanings as in Figure 10. From panels (a) and (b), the absolute differences in the wages and total time investment due to the recession are more considerable in the first several years and then fade later. In panels (c) and (d), I observe similar scarring effects in the simulation as in the data. The blue line is the benchmark for comparison, so it is 0 all the time. The initial wage loss is 10%, and the percentage change in wage declines over time and disappears. I define the scarring effect as disappearing when the percentage change in wage is smaller than 0.1%. In that case, the simulated scarring effect disappears in 15 years, consistent with the empirical finding. After the first 15 years, the wage profile during the recession seems to recover back to the normal path. From panel (d), on average, workers decrease total time investment on skills accumulation due to recession. However, the percentage decline in

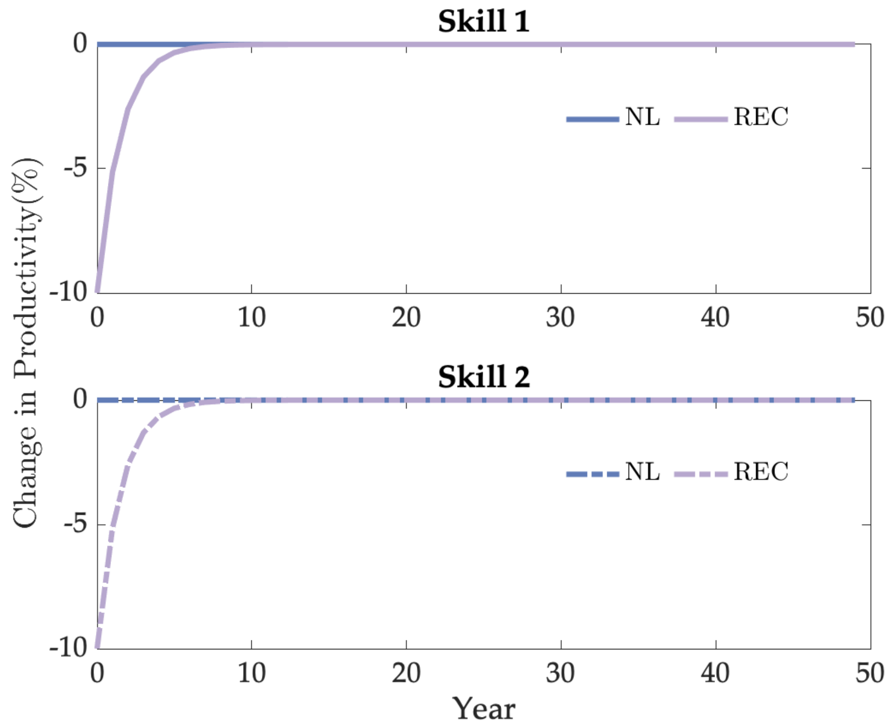


Figure 10: Change in Productivities Due to Recession

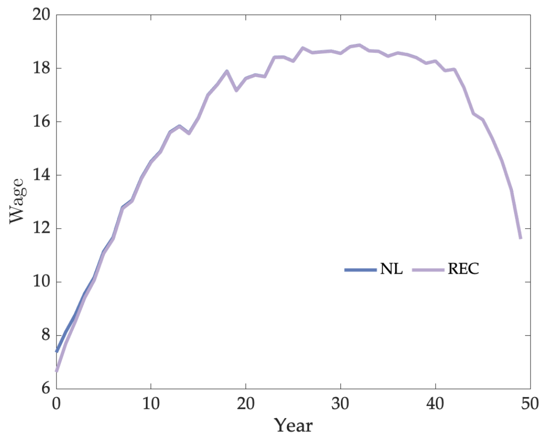
total time investment is smaller than the wage decline.

5.4 Scarring Effect in a Signal-Precise Economy

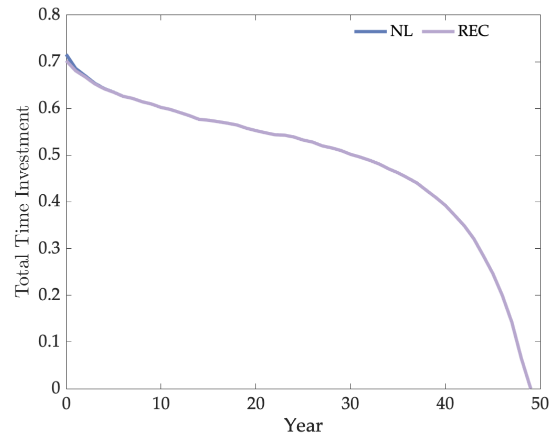
In this section, I analyze the impact of signals on the scarring effect by addressing: How does the scarring effect on wages differ in an economy with precise signals? To simulate the economy with more precise signals, I change the value of σ_ϵ from 1.60 to 0.01, keeping all other free parameters the same as in Table 7.

Figure 12 presents the simulation results in these two economies. The results for the economy with precise signals are denoted as "NL (Precise Signals)." In an economy where workers receive more accurate signals about true abilities in skill accumulation, workers, on average, allocate less time to skill accumulation but engage in more work, leading to a higher wage profile over the life cycle. Essentially, higher information friction intensifies workers' motivation to invest more in skill accumulation.

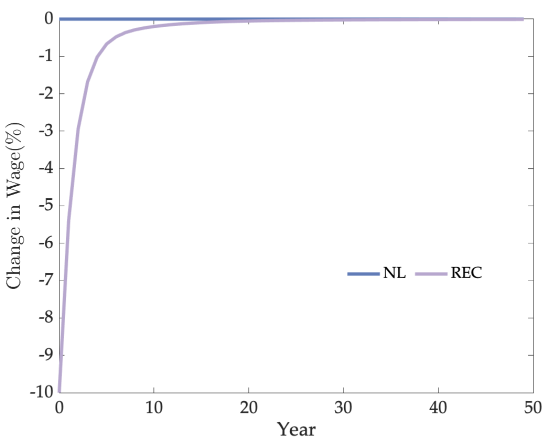
Next, I simulate a recession in the economy characterized by more precise signals, as detailed in section 5.3. Figure 13 depicts the percentage wage losses and the change in total time investment across the life cycle resulting from the recession in these two economies. With more precise signals, the impact of the scarring effect on wages persists for an extended duration, and the average magnitudes are larger. It occurs because more precise signals prevent workers from reducing their time investment significantly due to the recession.



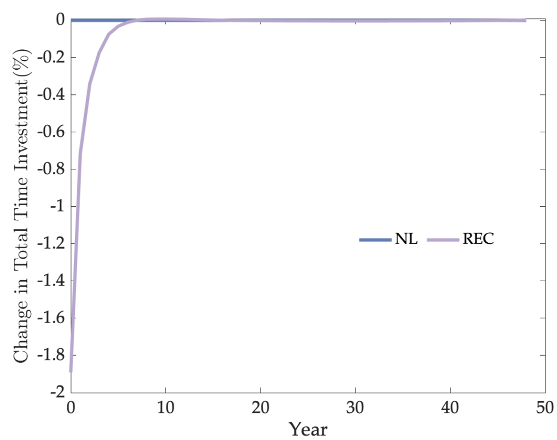
(a) Life-Cycle Wage



(b) Life-Cycle Total Time Investment



(c) Life-Cycle Wage Loss (%)



(d) Life-Cycle Total Time Investment Loss (%)

Figure 11: Changes Due to Recession

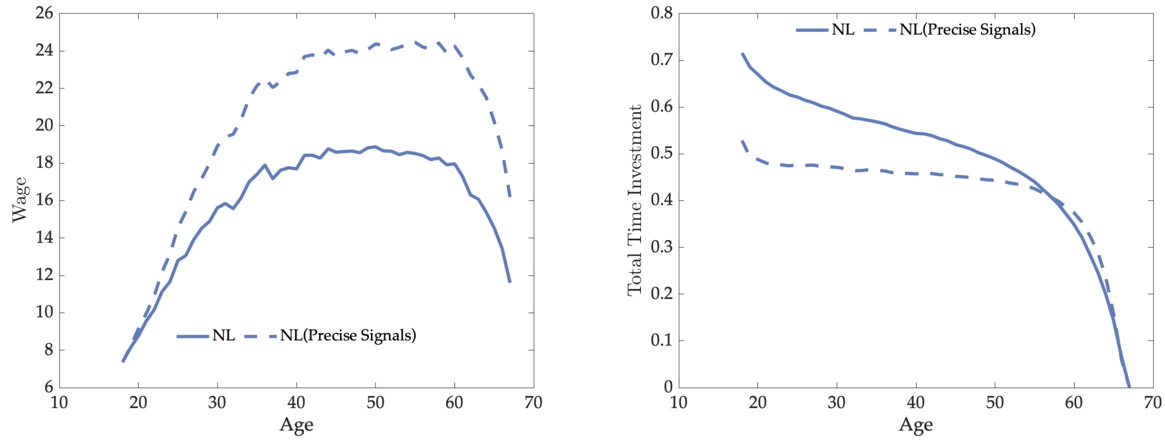


Figure 12: Hourly Wage (Left) & Total Time Investment (Right)

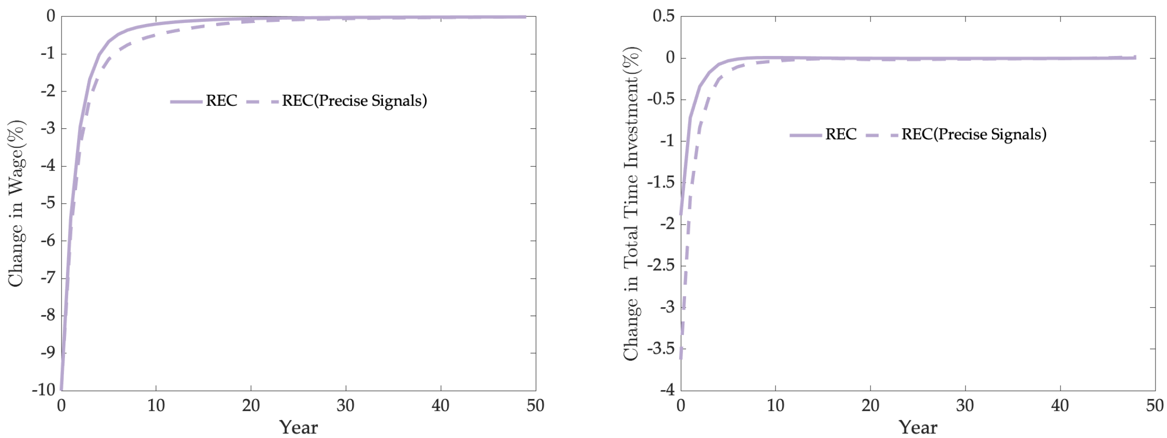


Figure 13: Change in Wage (%, Left) & Change in Total Time Investment (%, Right)

5.5 Decomposition of the Scarring Effect

So, what is the main reason that causes the persistent impact of a short-run shock on workers' wages? In this section, I decompose the contribution of each factor that leads to the scarring effect on wages.

The model presented in this paper captures two mechanisms that explain how a recession leads to lasting impacts on workers' wages. One reason is that, during a recession, the economy does not value skills as in normal times since the exogenous recovery process of productivity of skills follows Log-AR(1). The other reason is that workers, in response to the decline in skill productivity, endogenously reduce their time investment during a recession, resulting in long-term repercussions. I conduct a counterfactual scenario in which workers allocate the same amount of time to accumulate skills as they would when the economy is normal, but skill productivity remains at recession levels. The differences between the new counterfactual results and the paths under normal time capture the contribution of change in productivity. Similarly, the differences between the new counterfactual results and the paths under recession capture the contribution from

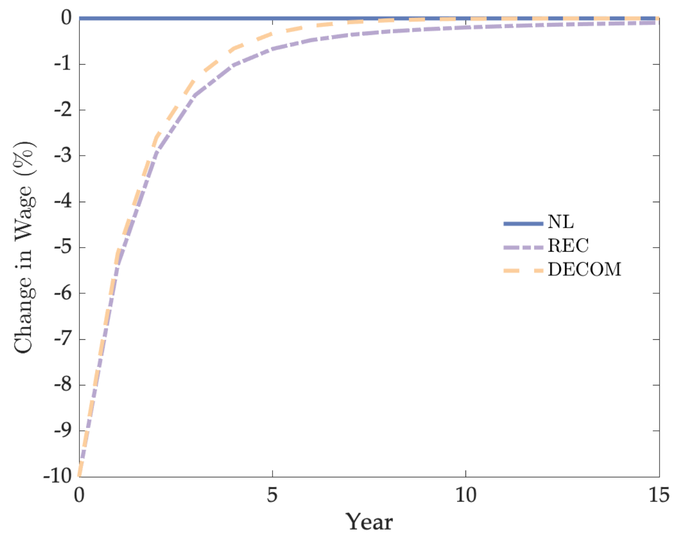


Figure 14: Decomposition of Scarring Effect on Wages

change in skill accumulation.

Figure 14 illustrates the average percentage change in wages over workers during the initial 15 years following the recession. The yellow line labeled "*DECOM*" also represents the simulated wage losses under the counterfactual scenario. In Figure 14, it is evident that the yellow line consistently remains slightly above the purple line, indicating that changes in the time investment in skills accumulation also contribute to wage losses over time. Furthermore, it is crucial to emphasize that while the contribution from the change in time investment to the persistent wage effect is less than that from the change in skill productivity in the first 8 years, the change in time investment becomes the sole factor responsible for prolonged wage scarring beyond the Year 7.

	Young Worker
Year 0-15	-1.26%
Change in Productivity	81.75%
Change in Skill Accumulation	18.25%
Year 0-7	-2.84%
Change in Productivity	89.79%
Change in Skill Accumulation	10.21%
Year 8-15	-0.20%
Change in Productivity	10.00%
Change in Skill Accumulation	90.00%

Table 9: Decomposition of Lifetime Wage Loss Due to Recession

I also calculate the average wage in present value attributed to the recession as a percentage per year during the initial 15 years following the recession. Then, I broke down the impact of each factor contributing to these losses. Table 9 presents the results of decomposition. In Table 9, the primary factor driving the decline in lifetime wages for young workers is the change in productivity during the recession recovery period. Surprisingly, despite the recession in this model being solely attributed to the decline in skills' productivity, the decrease in skill accumulation still accounts for roughly 20% of the decline in the remaining lifetime wage loss post-recession. Additionally, the decline in skill accumulation constitutes about 90% of the remaining lifetime wage loss eight years after the recession.

5.6 The Main Factor Causes Unequal Recovery Speed

The next question to address is why the same recession results in different recovery speeds among workers. The underlying reason must be related to the initial heterogeneity in workers' characteristics. In this section, I present the effect of the recession on the wages of heterogeneous workers in three dimensions: true abilities on skills accumulation ($\boldsymbol{\theta}$), initial signals about true abilities ($\hat{\boldsymbol{\theta}}_{\mathbf{0}}$), and initial stock of skills ($\boldsymbol{H_0}$). To demonstrate the significance of each dimension in elucidating heterogeneity in the recovery process, I compute the time-series average percentage wage losses among workers exhibiting heterogeneity in one dimension while averaging across the remaining two dimensions. Based on those data, I illuminate the importance of each dimension concerning the variation in recovery speed and the disparity in lifetime wage loss due to the recession, represented as the percentage decline in present-value wage per year. To ensure those results are comparable across different workers, I focus solely on the analysis from year 0 to year 15, corresponding to the duration of the average scarring effect on wages.

Based on panel (c) in Figure 11, the recovery speed is not consistent over time, displaying a percentage wage loss that appears to follow an exponential decay trend. The pattern exists for all types of workers. Thus, I denote the absolute value of the average wage loss at time t for workers in group i as $|\Delta \ln(w_t^i)|$, and it follows the following exponential function,

$$|\Delta \ln(w_t^i)| = |\Delta \ln(w_0^i)| exp\{-\lambda^i t\},$$

where $\lambda^i(\lambda^i > 0)$ is the decay constant. Then, I perform a half-life analysis on the wage loss decay to evaluate and discuss the recovery speed. The half-life (denoted as $t_{\frac{1}{2}}$) measures the number of years that require the percentage wage loss to reduce to half of its initial value. For an exponential decay, the half-life for each group of workers i can be computed as $t^i_{\frac{1}{2}} = \frac{\ln(2)}{\lambda^i}$. Thus, to discuss the recovery speed, I need first to estimate the decay constant λ^i for every group of workers. Drawing inspiration from the findings in section 5.5, I divide the recovery period into two segments. I estimate the decay constants that best fit the simulated wage losses separately from year 0 to year 7 and from year 8 to year 15.

For the average wage losses over all workers, decay constants are 0.59 and 0.16 for these two periods. Figure 15 demonstrates how accurately these exponential decay functions fit the simulated percentage wage loss from the model. The data suggests that, on average, it takes workers about 1.17 years (one year and two months) to halve the percentage wage losses from their initial level between year 0 and year 7. This duration increases to 4.26 years (4 years and 3 months) between year 8 and year 15.

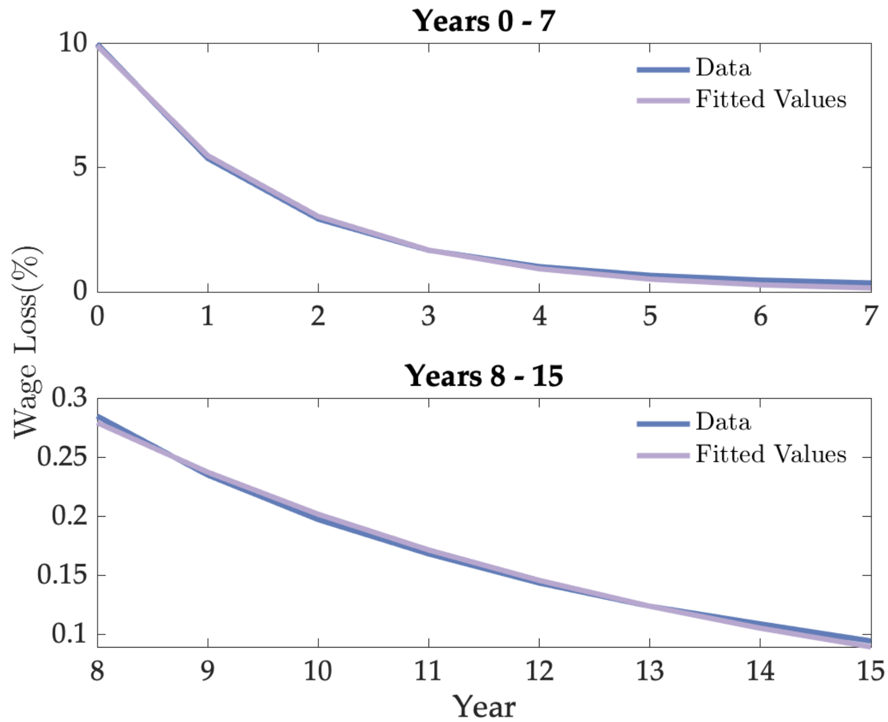
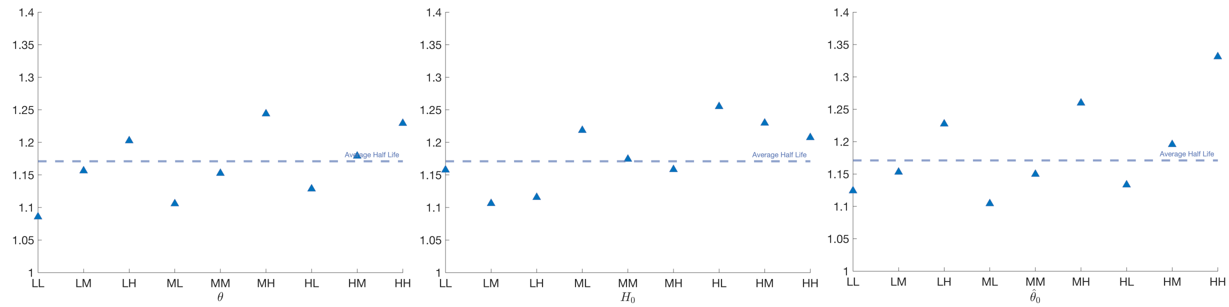


Figure 15: Estimated Exponential % Wage Loss Decay

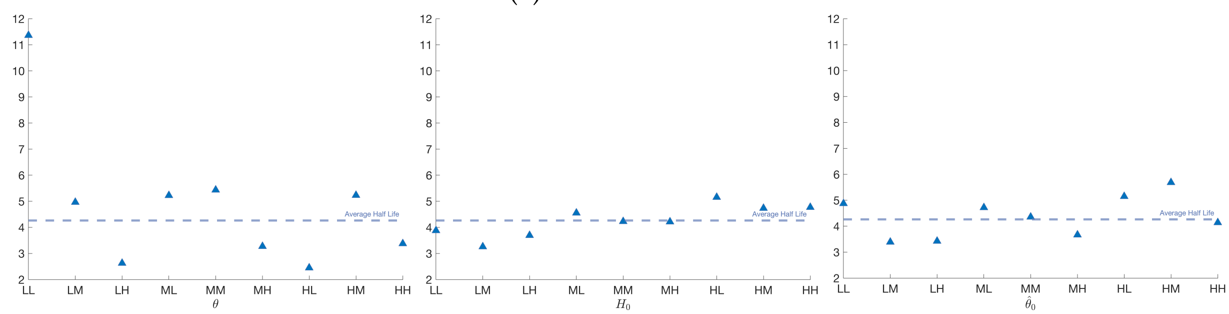
Then, I conduct the same analysis on the average wage losses across workers who differ in each of the three dimensions: true abilities on skills accumulation ($\boldsymbol{\theta}$), initial signals about true abilities ($\hat{\boldsymbol{\theta}}_{\mathbf{0}}$), and initial stock of skills ($\boldsymbol{H}_{\mathbf{0}}$). In Figure 16, the half-life for the average percentage wage loss decay is illustrated in the blue triangle for workers differing in one dimension while averaging across the remaining two dimensions. The blue dash lines are the average half-life for all workers mentioned in the previous paragraph. As mentioned in the previous sections, I discretize three states for each skill in each dimension, labeled $\{L, M, H\}$, representing the lowest, middle, and highest. For example, LL represents the worker with the lowest states on both skills, and LM means the lowest state in skill 1 and the middle state in skill 2.

Panel (a) presents results from year 0 to year 7, while panel (b) shows results from year 8 to year 15. The figures from left to right indicate the half-life for workers averaged across true skills accumulation abilities, initial stock of skills, and signals about true abilities. During the first period from year 0 to year 7, which marks the skills productivity recovery phase, minimal variation in the half-life is observed within each figure. However, among workers at the same level in skill 1, individuals with a comparative advantage in true abilities and signals related to skill 2 exhibit a slower recovery speed during this phase, attributable to the more considerable absolute decline in productivity of skill 2 compared to skill 1. In contrast, among workers at the same level in skill 1, those with the highest initial stock of skill 2 experience a faster recovery speed, as a higher initial stock of skill translates to higher investment costs in skill 2, prompting workers to shift their investment profile toward skill 1.

In panel (b), the variations among workers differing in each dimension are more pronounced than those from year 0 to year 7. Across the three dimensions of workers' heterogeneity, differences in true abilities in skills accumulation explain most of the variations in the recovery speed of workers. Notably, workers with the lowest true abilities in both skills accumulation have a half-life of around 11 years. Additionally, those workers who possess a comparative advantage in true abilities and signals related to skill 2 exhibit faster recovery speeds.



(a) Year 0 - Year 7



(b) Year 8 - Year 15

Figure 16: Half-Life for % Wage Loss Decay

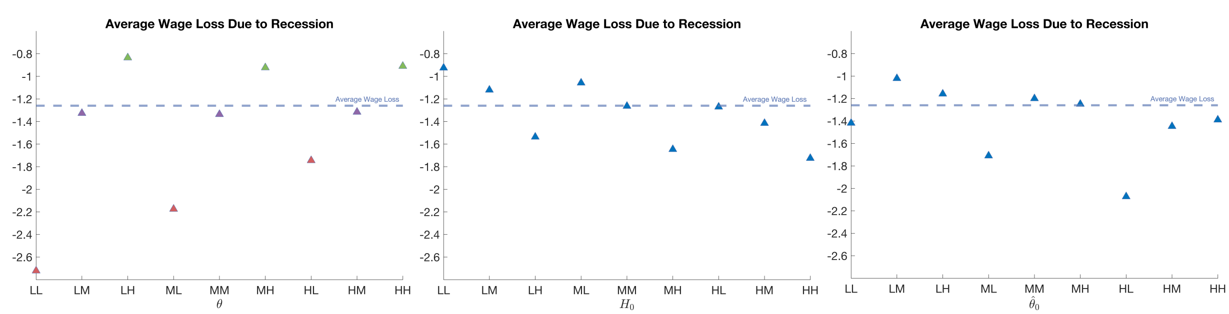


Figure 17: Average Lifetime Wage Losses Due to Recession

I reach the same conclusion when analyzing the variations in the average lifetime wage loss due to recession across workers differing in each dimension. To quantify the lifetime wage loss, I calculate the percentage decline in present-value wage every year for each worker. Figure 17 outlines the average lifetime wage losses for workers averaged across true skills accumulation abilities (left), the initial stock of skills (middle), and signals about true abilities (right). Consistently, across the three dimensions of workers' heterogeneity, differences in true abilities in skills accumulation primarily account for variations in average lifetime wage losses due to recession.

In the left figure, the red triangles represent the top three groups of workers experiencing the most substantial wage declines due to the recession. The purple triangles represent those facing moderate average wage losses, while the green triangles denote those least affected by the recession. Notably, the groups most severely impacted by the recession comprise workers with the lowest true ability in skill 2. Conversely, workers with a comparative advantage in accumulating skill 2 are least affected, given the smaller decline in investments in skill 2. Workers with the least true ability to accumulate both skills have the highest impact on wages due to the recession, experiencing approximately a 2.6% decline in wages every year.

6 Conclusion

The paper aims to quantify the main reason for the heterogeneity in the recovery speed of life-cycle wages across young workers with high school degrees only who graduated and entered the labor market during a recession. The existing empirical literature focuses on the heterogeneous recovery processes across workers categorized by different observed characteristics, such as educational levels and ages. In this study, utilizing the NLSY79 data while controlling for observed characteristics, I have identified substantial disparities in the wage recovery speed among workers in different residual wage quantiles. I find workers in higher residual wage quantiles have a faster speed of recovery after the recession. Notably, this paper is the first to document such findings, indicating that certain unobserved characteristics are essential in explaining this heterogeneity.

Thus, I establish a life-cycle multidimensional skills accumulation model with workers' idiosyncratic learning and endogenous occupational choices. I include these two features because they are critical in wage determinations throughout life. The model also consists of aggregate shocks in skill productivity for the recession simulation. Similar to Huggett et al. (2011) that accounts for factors causing the differences in age-wage profile, I assume workers are different in three dimensions: the true abilities bundle, the initial signals about true abilities, and the initial stocks of skills bundle. The model considers two factors that lead to persistent wage losses following a short-run recession. First, the economy does not place the same value on skills several years after the recession as during normal times. Second, workers endogenously reduce their time investments in skills accumulation due to the decline in the productivity of those skills. In the paper, I simulate the economy as in the early 1980s recession by initially reducing the productivity of both skills by 10% from year 0, which then follows a recovery process described by a log-AR(1) process. The recovery period for skill productivity spans 8 years, finishing at year 7. By year 15, workers, on average, return to normal wage levels. However, in an economy with more precise signals about workers' true abilities, the recovery process takes longer on average compared to the current conditions with higher information friction. The larger information friction motivates workers to invest more in skill accumulation, acting as a buffer during recessions. Additionally, in a counterfactual analysis, the reduction in time investment to skill accumulation explains about 20% of the lifetime wage losses due to the recession, rising to approximately 90% from year 8 to year 15. Finally, the heterogeneity in recovery speed is mainly due to the true abilities differences rather than the initial stock of skills and initial signal differences. These differences in true abilities result in distinct changes in time investments for skill accumulation.

There are several possible extensions of this paper. Firstly, the current focus is on the long-term consequences of graduating and entering the labor market during a recession on wages. Therefore, all the discussions in the paper are based on the condition of high school graduates who are employed during the recession. However, it is worth noting that there are related studies that examine the long-term costs of job losses in a recession (e.g., Davis and Von Wachter (2011), Huckfeldt (2022), Schmieder et al. (2023)). As an extension, I could enhance the current model by introducing an outside option of unemployment for workers, which would allow for considering transitions between employment and unemployment. Secondly, the current idiosyncratic learning process in the paper is only related to the number of working experiences. However, there are also papers documenting that firms (e.g., Wachter (2020)) or workers (e.g., Maclean and Hill (2015)) spend more time learning workers' true types than usual if they graduate in a recession. To account for this feature, I can also extend the current Bayesian learning process as a function of aggregate economic conditions.

References

Altonji, J. G., L. B. Kahn, and J. D. Speer (2016). Cashier or consultant? entry labor market conditions, field of study, and career success. *Journal of Labor Economics 34*(S1), S361–S401.

Browning, M., L. P. Hansen, and J. J. Heckman (1999). Micro data and general equilibrium models. *Handbook of macroeconomics 1*, 543–633.

Cavounidis, C. and K. Lang (2020). Ben-porath meets lazear: Microfoundations for dynamic skill formation. *Journal of Political Economy 128*(4), 1405–1435.

Cockx, B. and C. Ghirelli (2016). Scars of recessions in a rigid labor market. *Labour Economics 41*, 162–176.

Cortes, G. M., N. Jaimovich, and H. E. Siu (2017). Disappearing routine jobs: Who, how, and why? *Journal of Monetary Economics 91*, 69–87.

Cortes, G. M., N. Jaimovich, and H. E. Siu (2023). The growing importance of social tasks in high-paying occupations: Implications for sorting. *Journal of Human Resources 58*(5), 1429–1451.

Davis, S. J. and T. M. Von Wachter (2011). Recessions and the cost of job loss. Technical report, National Bureau of Economic Research.

Genda, Y., A. Kondo, and S. Ohta (2010). Long-term effects of a recession at labor market entry in japan and the united states. *Journal of Human resources 45*(1), 157–196.

Glover, A., J. Heathcote, D. Krueger, and J.-V. Ríos-Rull (2020). Intergenerational redistribution in the great recession. *Journal of Political Economy 128*(10), 3730–3778.

Guvenen, F., B. Kuruscu, S. Tanaka, and D. Wiczer (2020). Multidimensional skill mismatch. *American Economic Journal: Macroeconomics 12*(1), 210–44.

Heckman, J. J. (1979). Sample selection bias as a specification error. *Econometrica: Journal of the econometric society*, 153–161.

Huckfeldt, C. (2022). Understanding the scarring effect of recessions. *American Economic Review 112*(4), 1273–1310.

Huggett, M., G. Ventura, and A. Yaron (2011). Sources of lifetime inequality. *American Economic Review 101*(7), 2923–54.

Kahn, L. B. (2010). The long-term labor market consequences of graduating from college in a bad economy. *Labour economics 17*(2), 303–316.

Kambourov, G. and I. Manovskii (2008). Rising occupational and industry mobility in the united states: 1968–97. *International Economic Review 49*(1), 41–79.

Lazear, E. P. (2009). Firm-specific human capital: A skill-weights approach. *Journal of political economy 117*(5), 914–940.

Liu, K., K. G. Salvanes, and E. Ø. Sørensen (2016). Good skills in bad times: Cyclical skill mismatch and the long-term effects of graduating in a recession. *European Economic Review 84*, 3–17.

Maclean, J. C. and T. D. Hill (2015). Leaving school in an economic downturn and self-esteem across early and middle adulthood. *Labour Economics 37*, 1–12.

Neal, D. (1999). The complexity of job mobility among young men. *Journal of labor Economics 17*(2), 237–261.

Oreopoulos, P., T. Von Wachter, and A. Heisz (2012). The short-and long-term career effects of graduating in a recession. *American Economic Journal: Applied Economics 4*(1), 1–29.

Oyer, P. (2006). Initial labor market conditions and long-term outcomes for economists. *Journal of Economic Perspectives 20*(3), 143–160.

Pavan, R. (2011). Career choice and wage growth. *Journal of Labor Economics 29*(3), 549–587.

Rinz, K. (2022). Did timing matter? life cycle differences in effects of exposure to the great recession. *Journal of Labor Economics 40*(3), 703–735.

Roy, A. D. (1951). Some thoughts on the distribution of earnings. *Oxford economic papers 3*(2), 135–146.

Schmieder, J. F., T. Von Wachter, and J. Heining (2023). The costs of job displacement over the business cycle and its sources: evidence from germany. *American Economic Review 113*(5), 1208–1254.

Schwandt, H. and T. Von Wachter (2019). Unlucky cohorts: Estimating the long-term effects of entering the labor market in a recession in large cross-sectional data sets. *Journal of Labor Economics 37*(S1), S161–S198.

Speer, J. D. (2016). Wages, hours, and the school-to-work transition: the consequences of leaving school in a recession for less-educated men. *The BE Journal of Economic Analysis & Policy 16*(1), 97–124.

Umkehrer, M. (2019). Heterogenous effects of entering the labor market during a recession—new evidence from germany. *CESifo Economic Studies 65*(2), 177–203.

Von Wachter, T. and S. Bender (2008). Do initial conditions persist between firms? an analysis of firm-entry cohort effects and job losers using matched employer-employee data. In *The Analysis of Firms and Employees: quantitative and qualitative Approaches*, pp. 135–162. University of Chicago Press.

Wachter, T. V. (2020). The persistent effects of initial labor market conditions for young adults and their sources. *Journal of Economic Perspectives 34*(4), 168–194.

Yagan, D. (2019). Employment hysteresis from the great recession. *Journal of Political Economy 127*(5), 2505–2558.

Additional Tables

7 ASVAB Components	
Arithmetic Reasoning	Test the ability to transfer real-life problems into mathematical terms.
Mathematics Knowledge	Most of the questions concern subjects that are typically introduced in high school courses, such as geometry, algebra, and trigonometry.
Word Knowledge	A vocabulary test.
Paragraph Comprehension	Measure how well you can acquire information from written passages.
General Science	Measures basic factual knowledge taught in secondary school general science coyrses. Items are drawn from biology, medicine, chemistry, and physics.
Mechanical Comprehension	Visualize how built around basic machines (such as pulleys, levers, gears, and wedges) would work together.
Electronics Information	Measure knowledge of electrical terms. Check the familiarity with electrical equipment and the ability to solve electrical problems.

Table 10: Definitions of *ASVAB* Components from NLSY ATTACHMENT 106

Table 11: 26 *KSA* Skills Definitions

26 ONET KSA			
Number Facility	The ability to add, subtract, multiply, or divide quickly and correctly.	Engineering and Technology	Knowledge of the practical application of engineering science and technology. This includes applying principles, techniques, procedures, and equipment to the design and production of various goods and services.
Oral Comprehension	The ability to listen and understand information and ideas presented through spoken words and sentences.	Installation	Installing equipment, machines, wiring, or programs to meet specifications.
Mechanical	Knowledge of machines and tools, including their designs, uses, repair, and maintenance.	Chemistry	Knowledge of the chemical composition, structure, and properties of substances and of the chemical processes and transformations that they undergo. This includes uses of chemicals and their interactions, danger signs, production techniques, and disposal methods.
Mathematics Knowledge	Knowledge of arithmetic, algebra, geometry, calculus, statistics, and their applications.	Troubleshooting	Determining causes of operating errors and deciding what to do about it.
Biology	Knowledge of plant and animal organisms, their tissues, cells, functions, interdependencies, and interactions with each other and the environment.	Equipment Selection	Determining the kind of tools and equipment needed to do a job.
Deductive Reasoning	The ability to apply general rules to specific problems to produce answers that make sense.	Physics	Knowledge and prediction of physical principles, laws, their interrelationships, and applications to understanding fluid, material, and atmospheric dynamics, and mechanical, electrical, atomic and sub-atomic structures and processes.

Table 11 continued from previous page

26 ONET KSA			
Technology Design	Generating or adapting equipment and technology to serve user needs.	Mathematics Skills	Using mathematics to solve problems.
Mathematical Reasoning	The ability to choose the right mathematical methods of formulas to solve a problem.	Equipment Maintenance	Performing routine maintenance on equipment and determining when and what kind of maintenance is needed.
English Language	Knowledge of the structure and content of the English language including the meaning and the spelling of words, rules of composition, and grammar.	Repairing	Repairing machines or systems using the needed tools.
Inductive Reasoning	The ability to combine pieces of information to form general rules or conclusions (includes finding a relationship among seemingly unrelated events).	Information Ordering	The ability to arrange things or actions in a certain order or pattern according to a specific rule or set of rules (e.g., patterns of numbers, letters, words, pictures, mathematical operations).
Science	Using scientific rules and methods to solve problems.	Reading Comprehension	Understanding written sentences and paragraphs in work-related documents.
Written Comprehension	The ability to read and understand information and ideas presented in writing.	Building and Construction	Knowledge of materials, methods, and the tools involved in the construction or repair of houses, buildings, or other structures such as highways and roads.
Computers and Electronics	Knowledge of circuit boards, processors, chips, electronic equipment, and computer hardware and software, including applications and programming.	Operation and Control	Controlling operations of equipment or systems

	Arithmetic Reasoning	Mathematics Knowledge	Word Knowledge	Paragraph Comprehension	General Science	Mechanical Comprehension	Electronics Information
Number Facility	0.08	0.08	0.03	0.04	0.05	0.04	0.04
Oral Comprehension	0.04	0.04	0.07	0.07	0.04	0.03	0.03
Mechanical	0.02	0.02	0.02	0.02	0.03	0.06	0.03
Mathematics Knowledge	0.08	0.08	0.03	0.03	0.04	0.04	0.04
Biology	0.02	0.02	0.02	0.02	0.07	0.01	0.01
Deductive Reasoning	0.06	0.06	0.05	0.06	0.06	0.05	0.05
Technology Design	0.02	0.02	0.02	0.02	0.03	0.05	0.05
Mathematical Reasoning	0.08	0.08	0.03	0.04	0.04	0.04	0.04
English Language	0.03	0.04	0.10	0.09	0.02	0.02	0.02
Inductive Reasoning	0.05	0.06	0.05	0.08	0.05	0.04	0.04
Science	0.05	0.06	0.03	0.04	0.07	0.04	0.04
Written Comprehension	0.06	0.05	0.10	0.10	0.05	0.03	0.04
Computers and Electronics	0.03	0.03	0.03	0.03	0.02	0.03	0.07
Engineering and Technology	0.02	0.03	0.02	0.03	0.04	0.05	0.05
Installation	0.02	0.02	0.03	0.02	0.02	0.05	0.05
Chemistry	0.02	0.02	0.02	0.02	0.06	0.02	0.02
Troubleshooting	0.02	0.02	0.03	0.02	0.03	0.05	0.05
Equipment Selection	0.02	0.02	0.02	0.02	0.02	0.05	0.04
Physics	0.03	0.03	0.02	0.02	0.07	0.05	0.04
Mathematics Skills	0.09	0.08	0.03	0.03	0.04	0.04	0.04
Equipment Maintenance	0.01	0.02	0.02	0.02	0.02	0.04	0.04
Repairing	0.02	0.02	0.02	0.02	0.01	0.04	0.04
Information Ordering	0.06	0.06	0.06	0.06	0.04	0.03	0.04
Reading Comprehension	0.03	0.03	0.10	0.09	0.04	0.03	0.03
Building and Construction	0.02	0.02	0.02	0.02	0.01	0.04	0.02
Operation and Control	0.02	0.02	0.02	0.02	0.05	0.05	0.05

Table 12: Normalized *KSA-ASVAB* Skills Crosswalk Table

A Appendices

A.1 Data Appendix

In this paper, I follow the approach by Guvenen et al. (2020) to construct the annual panel data based on NLSY79's Work History Data File from 1979 to 2018. The work history data file contains weekly employment histories for up to five jobs. I keep a track of the weekly job index and construct the weekly hours worked in each job. In each week, I define the job that has been worked the highest number of hours as the main job and I record the number of hours worked in the main job. Then, I generate the yearly job index by obtaining the job index which individuals work the most in the given year. I keep only the record of the main job and then merge with other yearly reported demographic information and detailed information of employment such as occupation, industry, and hourly wage.

Occupation and Industry Codes: The process is the same as in Guvenen et al. (2020). To create consistent occupational codes and titles across the years, I first convert all occupational codes into the Census 1990 Three-Digit Occupation Code before the cleaning process. Similarly, I convert all industrial codes to 1-digit Census 1970 codes. NLSY79 provides each individual's employment information for five jobs in a given year. Notice that jobs are not the same as occupations because a job is attached to employers/firms. Thus, workers who work in the same occupation and industry under different employers are regarded as working in different jobs. Since some of the respondents in NLSY79 also participate in the CPS annual survey, I first use CPS information on occupation and industry if they do not report in the NLSY79 interviews. Then, I compute the employment spell for each job and determine the occupation title and corresponding industry most often observed during the spell. Similar to Kambourov and Manovskii (2008), promotions are eliminated in this paper since assuming the occupational switch does not happen without changing employers. Finally, I combine the weekly main job index with each job's occupational code, occupational title, and industrial code to obtain the worker's main occupation code, occupation title, industry and hourly wages.

Cognitive and Non-Cognitive Occupations: I apply the definitions used in Cortes et al. (2017) to categorize occupations into two groups. I define cognitive occupations are the combinations of non-routine cognitive, and non-cognitive occupations as the rest. The detailed classifications of occupations are listed in Table A1.

Gross Occupational Mobility over Life Cycle: In Figure 18, I analyze and compare the life-cycle occupational switching patterns of workers who graduate and enter the labor market either before (blue-solid line) or during (black-dashed line) the early 1980s recession. Surprisingly, I find no significant differences in the occupational switching behavior between these two entry groups. As a result, my paper primarily focuses on the generic probability of occupational switching. To calculate the generic probability of switching occupations across different age groups, I compute the gross occupational mobility. This is done by dividing the total number of occupational switches within each age group by the total number of workers within that age group. It's important to note that in the data set, workers of the same ages are not necessarily in the same time period because they

Cognitive Occupation	
Non-Routine Cognitive: Managerial and Professional Specialty Occupations (3-199), Technicians and Related Support Occupations (203-225), Broadcast Equipment Operators (228), Computer Programmers (229), Legal Assistants (234), Technician nec (235)	*Routine Cognitive:* Sales Occupations (243-290), Administrative Support Occupations, Including Clerical (303-391)
Non-Cogitive Occupation	
Routine Manual: Airplane Pilots (226,227, 233), Tool Programmers, Numerical Control (233), Precision Production, Craft, and Repair Occupations (503-699), Operators, Fabricators, and Laborers (703-890)	*Non-Routine Manual:* Service Occupations (403-469)

Table A1: Titles and Census 1990 Codes For Cognitive and Non-Cognitive Occupations

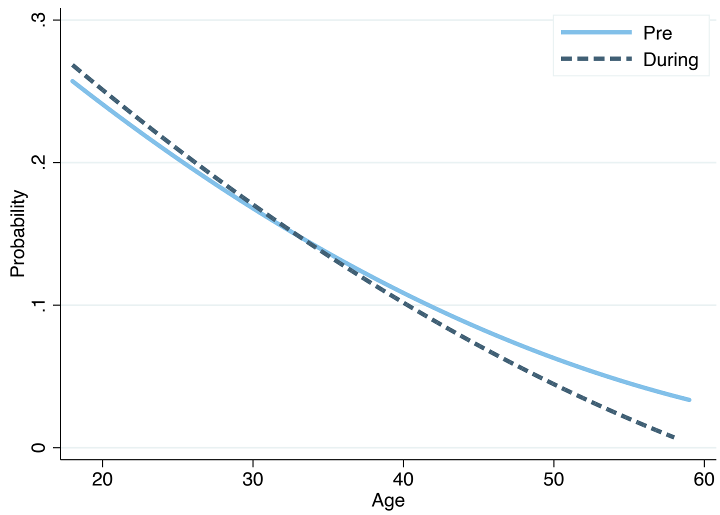


Figure 18: Probability of Switching Occupations Across Ages

may have responded to the first survey in 1979 at different ages. To isolate the age effect and eliminate time-related influences, I conduct a time-series logistic regression. This regression includes quadratic time dummies and the time-series unemployment rate as controls to estimate the age effect on gross occupational mobility.

Wage Information: NLSY79 contains information on the hourly rate of paid salaries or wages of five jobs in each interview. As discussed above, I only focus on the salaries/wages of the main job. All hourly rates of pay are deflated by Personal Consumption Expenditures Price Index (PCE) in real terms in 2000 dollars. I drop observations if the real wage is below $1 or higher than $1,000.

Sample Selection: My sample selection procedure is similar to Guvenen et al. (2020). I consider individuals who enter the labor market after receiving degrees; therefore, observations before receiving the highest degree are eliminated. Some individuals quit the job market and go back to school during the survey period, so I drop those observations, too.

Also, I limit our sample not to include individuals who started careers before the survey began. I define the labor market entry as when workers start to work more than 1200 hours (i.e., full-time working hours) in two consecutive periods across interviews. Besides, I exclude individuals who worked in the military services for more than two years and observations of individuals in the military force. I also eliminate observations after the last period a person reports a job. In addition, I consider workers who are weekly attached to the labor market by following the definitions in Guvenen et al. (2020) [12]. Thus, I guarantee my sample only contains workers who leave school for a while and are strongly attached to the labor market. I maintain observations in our sample that have valid occupation codes, industry codes, and wage information. I also restrict the sample to obtain valid O*NET-ASVAB information. Moreover, I eliminate multiple entries for each individual in a given period. For individuals with double entries, I drop the first observation in the year if it is the same as the previous year and the last observation if it is the same as the job in the next year. Then, I eliminate the multiple entries by keeping the first entry every year. Then, I drop the military sample and restrict the sample only to include the high school graduates. The summary of the remaining individuals and observations in the sample are listed in Table A2.

[12] In Guvenen et al. (2020), they consider workers who have been out of the labor market multiple times before working for at least ten years after starting their career. They discard only observations if an individual has worked for more than ten years before the first dropout or has been out of the labor market for only one year during the whole survey period.

	Number of Individuals	Number of Observations
Original Sample	12,686	699,301
Drop observations before if quit for receiving education and obtaining degree	12,686	698,245
Drop individuals who started career before 1979	11,022	603,258
Keep Observations when individuals work more than 1,200 hours in two consective peirods	9,640	467,305
Drop individuals who are in military force for more than 2 years	9,514	461,076
Drop observations worked in the military force	9,514	461,004
Drop observations when workers leave the job market and go back to school	9,514	460,286
Drop observations after the last time they worked	9,514	359,377
Drop individuals who are weakly attached to the labor market.	8,093	277,414
Eliminate double entries similar information from consecutive year	8,093	195,482
Drop observations if there is no occupation information	8,092	192,850
Keep valid nominal wage information	8,082	184,372
Keep observations from age between 16-60 with valid ASVAB scores	7,710	176,983
Keep valid industrial code and O*NET information	7,708	175,926
Drop multiple entries	7,708	164,171
Drop the military sample	7,140	159,165
Drop individuals without graduation and enrollment information	6,158	140,884
Drop observations before degree completion	6,117	127,822
Keep real wage between $1 and $1,000	6,112	126,942
Keep individuals who are employed in the year when they graduate	2,795	55,520
Keep high school graduates only	1,053	24,199

Table A2: Sample Selection from NLSY79, 1979-2018

A.2 Scarring Effects: Changing Recovery Speed

A.2.1 The Average Scarring Effect Across Workers

This section examines the effect of graduating and entering the labor market during a recession on the lifetime log wages of high school graduates. Given that my data set comprises individuals between 14 and 22 years old during the initial survey in 1979, I focus on the early 1980s recession, as it aligns with the young age of most individuals in my sample at that time. To address potential variations in the speed of recovery over time, I introduce an additional interaction variable into the baseline regression (equation 14). This variable is calculated as the product of each individual-specific unemployment rate upon entering the labor market and the square of their potential work experience over time. The main regression is below.

$$\begin{aligned}\ln(wage_{it}) = & \zeta_0 + \zeta_1 UE_i + \zeta_2 UE_i \times Exp_{it} + \zeta_3 UE_i \times Exp_{it}^2 + \zeta_4 Exp_{it} + \zeta_5 Exp_{it}^2 \\ & + \gamma^{'} Y_t + \Psi^{'} IA_i + u_{it},\end{aligned} \tag{14}$$

Under this nonlinear case, there are three relevant estimators for the scarring effect discussion. Similar to the linear case, ζ_1 represents the initial percentage wage loss for a one percentage point increase in the unemployment rate when an individual graduates from high school and enters the labor market. However, the recovery speed is determined jointly by ζ_2 and ζ_3. Specifically, the life-cycle wage profile of high school graduates exhibits a declining recovery speed if ζ_2 is positive and ζ_3 is negative. Conversely, the recovery speed increases if ζ_2 is positive and ζ_3 is positive. A positive ζ_2 and zero ζ_3 imply a constant recovery speed. For all other combinations of ζ_2 and ζ_3, high school graduates do not fully recover to their original life-cycle wage pattern. Table A3 lists these three estimates in panel A and fitted percentage wage losses for selected years of experience in panel B. The four columns in Table A3 list results with different additional control variables.

Table A3 highlights the persistent impact of graduating and entering the labor market during a recession on the life-cycle wages of high school graduates. In the baseline model, there is an initial wage loss of approximately 6% per one-percentage-point increase in the entry unemployment rate, with statistical significance at the 1% level. Furthermore, the recovery rate decelerates over time, and this effect remains statistically significant even after a decade. Similar to the results obtained in the linear case, the findings remain broadly consistent when additionally controlling for race and gender. However, when incorporating controls related to workers' abilities, the initial wage loss percentage decreases, leading to a shorter recovery period. Notably, the recovery speeds are approximately consistent across different sets of controls. Figure 19 plots the fitted percentage loss in wage over years of experience. I conclude that the effect of a one-percentage-point increase in the unemployment rate when graduating and entering the labor market on wages declines over time and vanishes about in 10 to 15 years.

	(1)	(2)	(3)	(4)
A: Regression coefficients				
UE	-0.0621***	-0.0596***	-0.0511***	-0.0475***
	[0.011]	[0.009]	[0.009]	[0.009]
$UE \times Exp$	0.0059***	0.0058***	0.0055***	0.0054***
	[0.001]	[0.002]	[0.002]	[0.002]
$UE \times Exp^2$	-0.00013***	-0.00012**	-0.00012**	-0.00011**
	[0.000]	[0.000]	[0.000]	[0.000]
B: Fitted effects for selected years of experience				
1	-0.0564***	-0.0539***	-0.0456***	-0.0422***
	[0.010]	[0.008]	[0.008]	[0.008]
4	-0.0407***	-0.0384***	-0.0308***	-0.0276***
	[0.008]	[0.007]	[0.007]	[0.007]
7	-0.0273***	-0.0251***	-0.0180**	-0.0151**
	[0.008]	[0.007]	[0.007]	[0.007]
10	-0.0162**	-0.0140*	-0.0074	-0.0045
	[0.008]	[0.008]	[0.008]	[0.008]
Controls:				
Gender & Race		X	X	X
Entry Occupational & Industrial Groups			X	X
ASVAB Test Scores				X
Obs.	24199	24199	24199	24199
Adjusted R-squared	0.240	0.272	0.306	0.319

Clustered standard errors in brackets
* p<0.10, ** p<0.05, *** p<0.01

Table A3: OLS Regression Results: Nonlinear

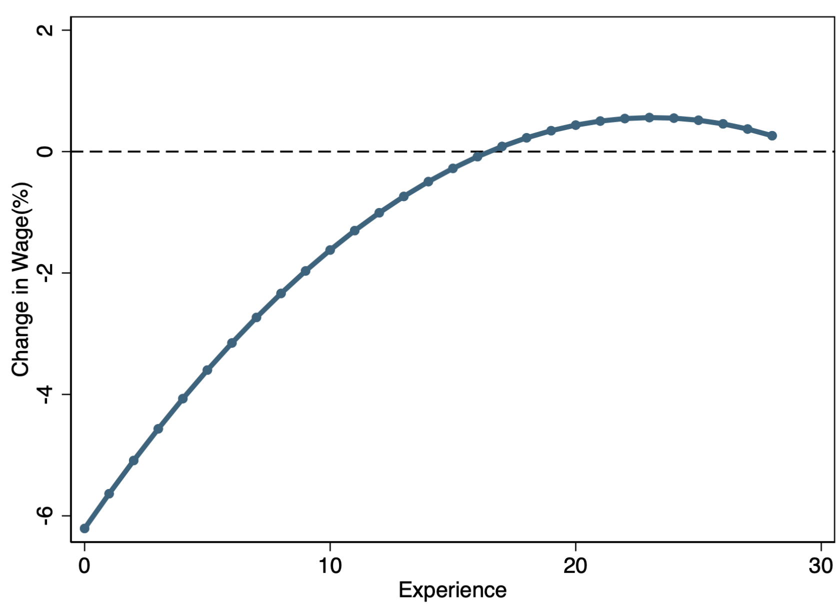


Figure 19: Fitted Wage Loss(%) Over Years of Experiences

A.2.2 The Heterogeneity in the Scarring Effect

In this section, I conduct quantile regression for workers in $25^{th}, 50^{th}$, and 75^{th} residual wage quantiles under different sets of controls and list those results in the Table A4. Compared to the results assuming a constant recovery speed, the scarring effect remains evident for workers in all residual wage quantiles. Furthermore, there is ongoing substantial heterogeneity in recovery trajectories among individuals. Notably, recovery processes maintain a constant speed for workers with medium residual wages. In contrast, those in the top and bottom 25^{th} residual wage groups experience a declining recovery speed. When considering controls for gender and race (see Column (2)), the results remain consistent with the baseline model (see Column (1)). Additionally, when accounting for workers' ability-related indicators commonly used in the literature, the initial percentage wage losses become more moderate. However, the recovery speeds do not exhibit significant variations due to these additional controls.

	(1)			(2)			(3)			(4)		
	Q(0.25)	Q(0.50)	Q(0.75)	Q(0.25)	Q(0.50)	Q(0.75)	Q(0.25)	Q(0.50)	Q(0.75)	Q(0.25)	Q(0.50)	Q(0.75)
A: Regression coefficients												
UE	-0.0446***	-0.0510***	-0.0605***	-0.0504***	-0.0504***	-0.0553***	-0.0383***	-0.0426***	-0.0476***	0.0347***	-0.0375***	-0.0473***
	[0.012]	[0.013]	[0.013]	[0.011]	[0.013]	[0.010]	[0.010]	[0.012]	[0.012]	[0.011]	[0.009]	[0.008]
$UE \times Exp$	0.0040**	0.0034*	0.0068***	0.0046**	0.0036*	0.0062***	0.0035*	0.0043*	0.0056***	0.0028	0.0040**	0.0058***
	[0.002]	[0.002]	[0.001]	[0.002]	[0.003]	[0.001]	[0.002]	[0.002]	[0.001]	[0.002]	[0.002]	[0.001]
$UE \times Exp^2$	-0.00010*	-0.00005	-0.00013***	-0.00010*	-0.00006	-0.00012***	-0.00007	-0.00008	-0.00011***	-0.00005	-0.00007	-0.00011***
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
B: Fitted effects for selected years of experience												
1	-0.0406***	-0.0477***	-0.0538***	-0.0458***	-0.0469***	-0.0492***	-0.0350***	-0.0384***	-0.0421***	-0.0319***	-0.0335***	-0.0416***
	[0.012]	[0.011]	[0.013]	[0.010]	[0.011]	[0.009]	[0.009]	[0.010]	[0.012]	[0.010]	[0.008]	[0.008]
4	-0.0300***	-0.0382***	-0.0353***	-0.0334***	-0.0370***	-0.0324***	-0.0256***	-0.0268***	-0.0270***	-0.0241***	-0.0226***	-0.0258***
	[0.010]	[0.009]	[0.011]	[0.009]	[0.007]	[0.008]	[0.008]	[0.007]	[0.011]	[0.008]	[0.006]	[0.008]
7	-0.0213**	-0.0296***	-0.0192**	-0.0228**	-0.0283***	-0.0177**	-0.0175*	-0.0166**	-0.0138	-0.0173*	-0.0131**	-0.0120
	[0.009]	[0.009]	[0.010]	[0.009]	[0.007]	[0.008]	[0.010]	[0.007]	[0.010]	[0.009]	[0.007]	[0.010]
10	-0.0144	-0.0218**	-0.0056	-0.0141	-0.0206**	-0.0052	-0.0107	-0.0078	-0.0025	-0.0113	-0.0048	-0.0002
	[0.010]	[0.010]	[0.010]	[0.011]	[0.009]	[0.009]	[0.011]	[0.009]	[0.010]	[0.011]	[0.008]	[0.010]
Gender & Race				X	X	X	X	X	X	X	X	X
Entry Occupational & Industrial Groups							X	X	X	X	X	X
ASVAB Test Scores										X	X	X
Adjusted R-squared	0.232	0.236	0.235	0.268	0.271	0.270	0.300	0.304	0.300	0.312	0.317	0.313

Clustered standard errors in brackets
* p<0.10, ** p<0.05, *** p<0.01

Table A4: Quantile Regression Results: Nonlinear

A.3 O*NET & Variables

I use the Occupational Information Network (O*NET) in this paper to obtain occupation-specific skills requirements information. O*NET provides hundreds of standardized occupational descriptors over thousands of occupations and their workers' attributes. O*NET Data Collection Program [13] sends out several questionnaires to occupational experts. Occupational experts are required to fill out questionnaires related to work activities, knowledge, and work context in each occupation and a background survey about workers' attributes. I use the 4.0 version of the O*NET analysts' database, the final version of an "analyst ratings only" O*NET database. This version does not include survey participants such as job incumbents, so it provides more consistent measurements across occupations[14]. The program continuously collects responses and updates the data set annually. The detailed descriptions of all variables and sample selections are explained below.

A.3.1 Linking Skill Measurements with NLSY79

O*NET contains two scale variables for detailed skills, knowledge, and abilities in 900 8-digit ONET-SOC occupations. The total number of skill, knowledge, and ability (*KSA* hereafter) descriptors for each occupation is 120. The participants in the survey are asked to rate the importance for each *KSA* descriptor from $[1, 5]$, measuring from "*Not Important*" to "*Extremely Important*." Suppose the participant does not rate the *KSA* as "*Not Important*" to perform the job. In that case, they need to answer the required level of this *KSA* descriptor to perform the job ranging from $[1, 7]$. For example, low-level deductive reasoning is to know that a stalled car can coast downhill, while high-level deductive reasoning is to design an aircraft wing using principles of aerodynamics. Thus, the meanings underlying the same scale of level are different across *KSA* descriptors. In this paper, I only focus on the importance of each *KSA* descriptor since it provides a consistent idea behind each scale number. I normalize the importance scale from $[0, 1]$.

I apply all 120 *KSA* skill descriptors and check the skills required in all occupations. Almost all occupations are frontier occupations since no occupation can dominate any others' requirements of skills in all 120 dimensions. Exceptions are *Crossing Guards and Flaggers*, *Janitors and Cleaners Except Maids and Housekeeping Cleaners*, *Tapers*, *Sewing Machine Operator*. Thus, the frontier occupation assumptions are not unacceptable, as shown in real data. Then, by utilizing *KSA* scale in all occupations, I can get an estimate of σ based on equation (3) by minimizing the sum of square errors. The estimated σ is around 3.2.

I obtain the mapping between O*NET occupational detailed *KSA* descriptors to coarser skill descriptors used in NLSY79 based on the technical summary from ASVAB Career Ex-

[13] For more information, check https://onet.rti.org/about.cfm.

[14] Starting from the 5.0 version, they also include sampled workers in establishments as survey participants. Workers sampled from establishments are randomly assigned to answer only one out of three questionnaires related to their occupations, such as work activities, knowledge, and work context. The establishment respondent is asked to complete a background survey. https://www.onetcenter.org/dictionary/27.3/excel/appendix_updates.html

ploration Program [15]. According to the technical summary report, the experts identified 26 O*NET descriptors most related to the 7 ASVAB test components. The definitions of 7 ASVAB components and 26 *KSA* descriptors are listed in Tables 10 and 11 separately. For each *ASVAB* component, I normalize the sum of conversion rates across 26 *KSA* skills into 1. The normalized *KSA-ASVAB* skills crosswalk table is listed in Table 12. I recalculate and get the weighted average scores across all *KSA* descriptors for each ASVAB component. I want to get as much occupational information as possible, so I use the sample before selecting only high school graduates. In total, that sample contains 299 unique occupations.

A.3.2 Occupational Intensities and Workers' of Initial Prior Beliefs on Skill Accumulating Abilities

In this paper, I only concentrate on measurable intellectual and unmeasureable skills. The measurable intellectual skill is the composite of the 7 ASVAB test components, and the unmeasurable skill is measured and computed based on the frontier occupation assumption. The unmeasurable skill can be interpreted as the composition of skills that are hard to measure in written tests, such as interpersonal and psychomotor skills. I denote the measurable intellectual skill as skill 1 ($n = 1$) and the unmeasurable skill as skill 2 ($n = 2$). I explain the detailed processes for obtaining the occupational requirements and workers' accumulation abilities under these two skill dimensions.

Occupational Intensity of Skills. I first start with the occupational intensity of skills $\boldsymbol{J}_m = (J_{m,1}, J_{m,2})$ for any occupation m. I extract the one-dimensional measured intellectual skill from these 7 ASVAB components by applying principle component analysis (PCA). The measured intellectual skill is the first principle component, and it can explain 90.57% of the total variance among the occupational requirements in the original 7 ASVAB components. The vector of component loadings for the measured intellectual skill is denoted as $\boldsymbol{L}^1 = (L^1_1, L^1_2, ..., L^1_7)$ and its multiplication with the initial intensity of 7 ASVAB components gives the intensity of skill 1. The vector values are listed in Table A5. Then, I calculate the percentage ranking by normalizing the computed occupational requirement of measured intellectual skill into $[0, 1]$ to satisfy the assumption in the theoretical model. Notice that the component loadings for measurable intellectual skills are only specific to ASVAB components.

Since I assume all the existing 299 occupations in our yearly panels are equilibrium outcomes, they are all frontier occupations based on Proposition 2. Then, from equation (3), the intensity of unmeasurable skill required in each occupation m is the residual

$$J_{m,2} = \left(1 - J^{\sigma}_{m,1}\right)^{\frac{1}{\sigma}}.$$

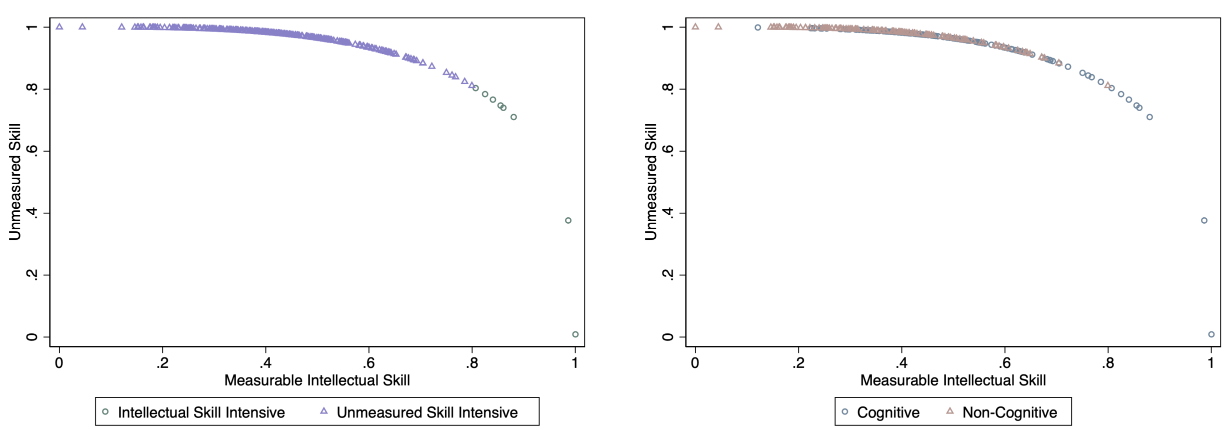
For each occupation m, I can back up the component loadings for the unmeasurable skill $\boldsymbol{L}^2_m = (L^2_{m,1}, L^2_{m,2}, ..., L^2_{m,7})$ by minimizing its Euclidean norm while holding its multiplication with the original intensity of 7 ASVAB components equals to the computed levels of

[15] ASVAB Career Exploration Program was established by the US Department of Defense to help high-school students predict future academic and career success.

Variables	Component Loadings (L^1)
Arithmetic Reasoning	0.3863
Mathematics Knowledge	0.3881
Word Knowledge	0.3789
Paragraph Composition	0.3796
General Science	0.3875
Mechanical Comprehension	0.3515
Electronics Information	0.3726

Table A5: Component Loadings in Measurable Intellectual Skill

skill 2 $J_{m,2}$. Thus, the unmeasurable skill's component loadings are occupation-ASVAB-component-specific. The plot of all frontier occupations in the yearly panels along the two skill dimensions is shown in Figure 20. Each point presents an occupation. From Figure 20, *Garbage Collectors* and *Housekeepers* are the occupations with the lowest measurable intellectual skill and highest unmeasurable skill. In contrast, *Chemical Engineers* and *Aerospace Engineers* are the occupations with the highest measurable intellectual skill and lowest unmeasurable skill. In panel (a), I categorize occupations based on whether they are intellectually intensive. From the model, an intellectual-skill-intensive occupation has requirement in skill 1 higher than $(0.5)^{\frac{1}{\sigma}}$. From the plot, most occupations in the data set are categorized as unmeasurable-skill-intensive groups. I also categorize occupations into cognitive and non-cognitive occupations following Cortes et al. (2017)'s definitions. I plot them in the two skills dimensions in panel (b). I see large overlapping parts in the middle of the plot. However, those occupations that mainly utilize the unmeasurable skill are non-cognitive. At the same time, those requiring extremely high intellectual skills are cognitive occupations.

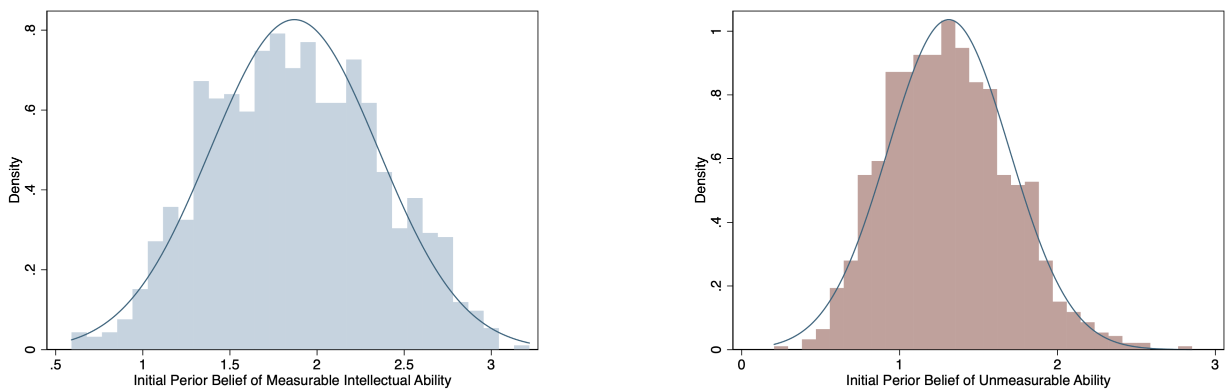


(a) Skill-Intensive Groups

(b) Cognitive and Non-cognitive

Figure 20: Frontier Occupations in 2-Skill Dimensions

Workers' Initial Prior Beliefs on Skill Accumulation Abilities: First, I calculate the accuracy of each worker's test scores in the 7 ASVAB components. Then, I use the loading components to transform the original seven ASVAB component test scores into two dimensions. Since the loading components for unmeasurable skills vary across occupations, I employ the component loadings of the worker's initial occupation from my yearly panels. Consequently, I derive each worker's initial prior beliefs about skill accumulation abilities, denoted as $\hat{\boldsymbol{\theta}}_0^i = (\hat{\theta}_{0,1}^i, \hat{\theta}_{0,2}^i)$, by multiplying the test accuracy scores and component loadings.

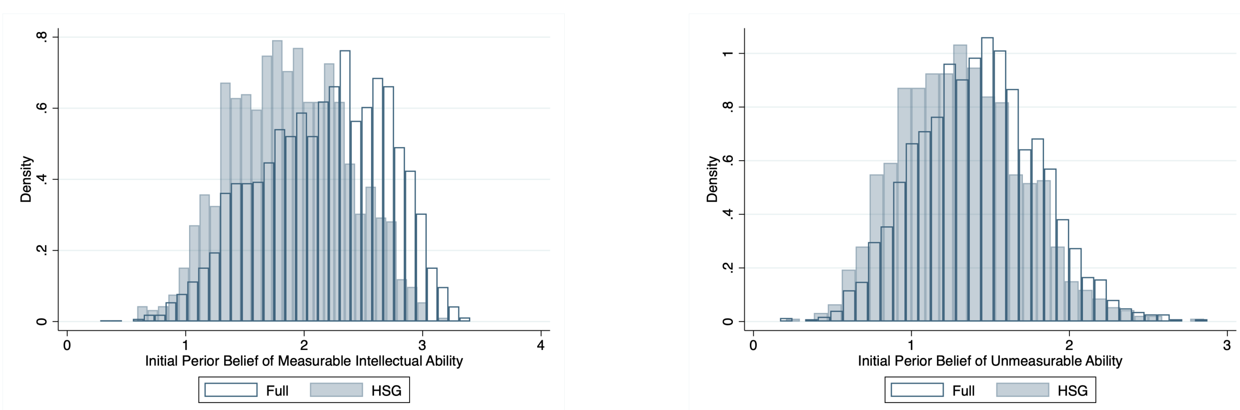


(a) Measurable Intellectual Ability

(b) Unmeasurable Ability

Figure 21: Distribution of Test Scores Among High School Graduates

The distributions of test scores for measurable intellectual and unmeasurable abilities among high school graduates in my sample are presented in Figure 21. These distributions closely resemble normal curves, as indicated by the solid curves representing the normal density functions for both abilities. On average, high school graduates tend to have higher prior beliefs about their measurable intellectual ability, with less variation among workers than their unmeasurable ability.



(a) Measurable Intellectual Ability

(b) Unmeasurable Ability

Figure 22: Comparisons of Distribution of Test Scores

Figure 22 displays the same distributions mentioned earlier, but this time for two different samples: one containing all educational degrees (labeled as "FULL") and the other consisting solely of high school graduates (labeled as "HSG"). The transparent histogram bars represent the distribution of the "FULL" sample, while the light blue-filled histogram bars correspond to the distribution of high school graduates, as seen previously in Figure 21. Interestingly, I observed that the distributions of prior beliefs for the sample with workers of all educational backgrounds are skewed to the right compared to those for high school graduates. Furthermore, the variances in the distributions of prior beliefs about true abilities on skill accumulations are smaller for the high school graduate sample when compared to the sample, including workers with all educational degrees.

A.4 Theoretical Proofs

A.4.1 Proof to Result 1

Worker i's prior belief in time t is given as ($t = 1, ..., T$)

$$\begin{aligned}
\hat{\theta}^i_{t,n} &= \frac{\lambda_{t-1,n}}{\lambda_{t,n}}\left[\frac{\lambda_{t-2,n}}{\lambda_{t-1,n}}\hat{\theta}^i_{t-2,n} + \frac{\lambda_{\epsilon_n}}{\lambda_{t-1,n}}(\theta^i_n + \epsilon^i_{t-2,n})\right] + \frac{\lambda_{\epsilon_n}}{\lambda_{t,n}}(\theta^i_n + \epsilon^i_{t-1,n}) \\
&= \frac{\lambda_{0,n}}{\lambda_{t,n}}\hat{\theta}^i_{0,n} + \frac{\lambda_{\epsilon_n}}{\lambda_{t,n}}t\theta^i_n + \frac{\lambda_{\epsilon_n}}{\lambda_{t,n}}\sum_{m=0}^{t-1}\epsilon^i_{m,n}
\end{aligned}$$

where $\lambda_{t,n} = \lambda_{t-1,n} + \lambda_{\epsilon_n} = \lambda_{0,n} + t\lambda_{\epsilon_n}$.

Since $\hat{\theta}^i_{0,n} = \theta^i_n + \tau_n$, I can simplify even further

$$\begin{aligned}
\hat{\theta}^i_{t,n} &= \frac{\lambda_{0,n}}{\lambda_{t,n}}(\theta^i_n + \tau_n) + \frac{\lambda_{\epsilon_n}}{\lambda_{t,n}}t\theta^i_n + \frac{\lambda_{\epsilon_n}}{\lambda_{t,n}}\sum_{m=0}^{t-1}\epsilon^i_{m,n} \\
&= (\frac{\lambda_{0,n}}{\lambda_{t,n}} + \frac{t\lambda_{\epsilon_n}}{\lambda_{t,n}})\theta^i_n + \frac{\lambda_{0,n}}{\lambda_{t,n}}\tau^i_n + \frac{\lambda_{\epsilon_n}}{\lambda_{t,n}}\sum_{m=0}^{t-1}\epsilon^i_{m,n} \\
&= \theta^i_n + \frac{\lambda_{0,n}}{\lambda_{t,n}}\tau^i_n + \frac{\lambda_{\epsilon_n}}{\lambda_{t,n}}\sum_{m=0}^{t-1}\epsilon^i_{m,n}
\end{aligned}$$

Thus, $\mathbb{E}(\hat{\theta}^i_{t,n} - \theta^i_n) = 0$,

$$\begin{aligned}
Var(\hat{\theta}^i_{t,n} - \theta^i_n) &= (\frac{\lambda_{0,n}}{\lambda_{t,n}})^2\sigma^2_{\tau_n} + (\frac{\lambda_{\epsilon_n}}{\lambda_{t,n}})^2 t\sigma^2_{\epsilon_n} \\
&= \frac{\lambda_{0,n}}{\lambda^2_{t,n}} + \frac{\lambda_{\epsilon_n}}{\lambda^2_{t,n}}t \\
&= \frac{1}{\lambda_{t,n}}
\end{aligned}$$

So $Var(\hat{\theta}^i_{t,n} - \theta^i_n)$ declines when time t grows.

A.4.2 Proof to Proposition 2

Suppose an occupation m is not one of the frontier occupations, and there exists a frontier occupation o with $J_{o,n} = J_{m,n}$ ($\forall n = 1, ..., N-1$) and $J_{o,N} = J_{m,N} + \xi(\xi > 0)$. Consider occupations m and o are competing for worker i with the stock of skill sets $\boldsymbol{H}^i_t$ at time t.

Occupation m offers the contract defined as $\boldsymbol{R}^*_{m,t} = (R^*_{m,t,1}, R^*_{m,t,2}, ..., R^*_{m,t,N})$ and gains profit

$$\pi^i_{m,t} = \sum_{n=1}^{N}\left[A_{t,n}J_{m,n}H^i_{t,n} - R^*_{m,t,n}H^i_{t,n}\right] \geq 0$$

from hiring worker i. Occupation o can always set $\boldsymbol{R}^*_{o,t} = (R^*_{o,t,1}, R^*_{o,t,2}, ..., R^*_{o,t,N})$ as $R^*_{o,t,n} =$

$R^*_{m,t,n}$ ($\forall n = 1, ..., N-1$) and $R^*_{o,t,N} = R^*_{m,t,N} + \varsigma(\varsigma > 0)$ to win the worker i from occupation m. By offering that contract for worker i, occupation o can cut off the profit of occupation m to 0, and the profit of occupation o is shown in the following equation,

$$
\begin{aligned}
\pi^i_{o,t} &= \sum_{n=1}^{N} \left[A_{t,n} J_{o,n} H^i_{t,n} - R^*_{o,t,n} H^i_{t,n} \right] \\
&= \sum_{n=1}^{N-1} A_{t,n} J_{m,n} H^i_{t,n} + A_{t,N} J_{m,N} H^i_{t,N} + A_{t,N} \xi H^i_{t,N} - \sum_{n=1}^{N-1} R^*_{m,t,n} H^i_{t,n} - R^*_{m,t,N} H^i_{t,N} - \varsigma H^i_{t,N} \\
&= \underbrace{\pi^i_{m,t}}_{=0} + \xi A_{t,N} H^i_{t,N} - \varsigma H^i_{t,N}.
\end{aligned}
$$

Thus, there always exist an frontier occupation o to offer $R^*_{m,t,n} = R^*_{o,t,n}$ ($\forall n = 1, ..., N-1$) and $R^*_{o,t,N} = R^*_{m,t,N} + \varsigma$ with $0 < \varsigma < \xi A_{t,N}$ to ensure occupation m fails in the hiring competition.

A.4.3 Proof to Proposition 3

Suppose one frontier occupation m offers $\boldsymbol{R}^*_{m,t} = (R^*_{m,t,1}, R^*_{m,t,2}, ..., R^*_{m,t,N})$ with $R^*_{m,t,n} = A_{t,n} J_{m,n} + \zeta_n$ to the worker i at time t. The profit of occupation m at time t if it successfully hires the worker i is

$$
\pi^i_{m,t} = \sum_{n=1}^{N} \left[A_{t,n} J_{m,n} H^i_{t,n} - R^*_{m,t,n} H^i_{t,n} \right] = -\sum_{n=1}^{N} \zeta_n H^i_{t,n}.
$$

Thus, $\sum_{n=1}^{N} \zeta_n H^i_{t,n} \leq 0$, and occupation m is willing to offer this contract and does not have a negative profit.

The posted wage for any worker i in occupation m at time t can be shown in the following expression,

$$
\begin{aligned}
w^i_{m,t} &= \sum_{n=1}^{N} R^*_{m,t,n} H^i_{t,n} \\
&= \sum_{n=1}^{N} (A_{t,n} J_{m,n} + \zeta_n) H^i_{t,n} \\
&= \sum_{n=1}^{N} A_{t,n} J_{m,n} H^i_{t,n} + \underbrace{\sum_{n=1}^{N} \zeta_n H^i_{t,n}}_{\leq 0}.
\end{aligned}
$$

To prove this proposition, I need to show there always exists another frontier occupation at time t that can offer better wages to the worker i if frontier occupation m offers $\boldsymbol{R}^*_{m,t}$. Consider another occupation o with $J_{o,1} = J_{m,1} + \frac{\sum_{n=1}^{N} \zeta_n H^i_{t,n}}{A_{t,1} H^i_{t,1}} \leq J_{m,1}$, $J_{o,n} = J_{m,n}$ ($\forall n = 2, 3, ..., N-1$), and $J_{o,N} = (1 - \sum_{n=1}^{N-1} J^{\sigma}_{o,n})^{\frac{1}{\sigma}} \geq J_{m,N}$. In occupation o, it offers $\boldsymbol{R}^*_{o,t}$ with $R^*_{o,t,n} = A_{t,n} J_{m,n}$ ($\forall n = 1, ..., N$) to the worker i at time t, and worker i gains higher wage from occupation o.

$$\begin{aligned}
w^i_{o,t} &= \sum_{n=1}^{N} R^*_{o,t,n} H^i_{t,n} \\
&= \sum_{n=1}^{N} A_{t,n} J_{o,n} H^i_{t,n} \\
&= A_{t,1} J_{o,1} H^i_{t,1} + \sum_{n=2}^{N-1} A_{t,n} J_{m,n} H^i_{t,n} + A_{t,N} J_{o,N} H^i_{t,N} \\
&= A_{t,1}\big(J_{m,1} + \frac{\sum_{n=1}^{N} \zeta_n H^i_{t,n}}{A_{t,1} H^i_{t,1}}\big) H^i_{t,1} + \sum_{n=2}^{N-1} A_{t,n} J_{m,n} H^i_{t,n} + A_{t,N} J_{o,N} H^i_{t,N} \\
&= A_{t,1} J_{m,1} H^i_{t,1} + \sum_{n=2}^{N-1} A_{t,n} J_{m,n} H^i_{t,n} + \sum_{n=1}^{N} \zeta_n H^i_{t,n} + A_{t,N} J_{o,N} H^i_{t,N} \\
&= w^i_{m,t} - A_{t,N} J_{m,N} H^i_{t,N} + A_{t,N} J_{o,N} H^i_{t,N} \geq w^i_{m,t}.
\end{aligned}$$

To win the competition, occupation m continues to decline its profit until $\zeta_n = 0 (\forall n = 1, ..., N)$ and its profit becomes 0.

A.4.4 Proof to Proposition 4

I solve the model from backward induction. In period T, $V_{T+1}(\boldsymbol{A_{T+1}}, \boldsymbol{H_{T+1}}, \boldsymbol{\hat{\theta}^i_{T+1}}) = 0$, the problem is

$$V_T(\boldsymbol{A_T}, \boldsymbol{H_T}, \boldsymbol{\hat{\theta}^i_T}) = \max_{\{\boldsymbol{q^i_T}, \boldsymbol{J^i}\}} \frac{\left[(1 - \sum_{n=1}^{N} q^i_{T,n}) \sum_{n=1}^{N} A_{T,n} J_n H^i_{T,n}\right]^{\eta} - 1}{\eta} + 0$$

$$s.t. \sum_{n=1}^{N} J_n^{\sigma} \leq 1, 0 \leq J^n \leq 1, \quad \forall n$$

$$\sum_{n=1}^{N} q^i_{T,n} \leq 1, 0 \leq q^i_{T,n} \leq 1, \quad \forall n$$

$$\boldsymbol{A_T}, \boldsymbol{H_T}, \boldsymbol{\hat{\theta}^i_T} \text{ are given.}$$

Denote $\mu_T, \phi_{T,n}, \phi'_{T,n}, \Phi_T, v_{T,n}, v'_{T,n}$ as multipliers. The Lagrangian is

$$\mathcal{L} = \frac{\left[(1 - \sum_{n=1}^{N} q^i_{T,n}) \sum_{n=1}^{N} A_{T,n} J_n H^i_{T,n}\right]^{\eta} - 1}{\eta} - \mu_T\big(\sum_{n=1}^{N} J_n^{\sigma} - 1\big) - \Phi_T\big(\sum_{n=1}^{N} q^i_{T,n} - 1\big)$$
$$+ \sum_{n=1}^{N} \left[\phi_{T,n} J_n + v_{T,n} q^i_{T,n} - \phi'_{T,n}(J_n - 1) - v'_{T,n}(q^i_{T,n} - 1)\right]$$

The first order conditions are

$$[J_n]:\left[(1-\sum_{n=1}^{N}q^i_{T,n})\sum_{n=1}^{N}A_{T,n}J^i_{T,n}H^i_{T,n}\right]^{\eta-1}(1-\sum_{n=1}^{N}q^i_{T,n})A_{T,n}H^i_{T,n}-\mu_T\sigma(J^i_{T,n})^{\sigma-1}+\phi_{T,n}-\phi^{'}_{T,n}=0,\quad\forall n$$

$$[q^i_{T,n}]:-\left[(1-\sum_{n=1}^{N}q^i_{T,n})\sum_{n=1}^{N}A_{T,n}J^i_{T,n}H^i_{T,n}\right]^{\eta-1}\sum_{n=1}^{N}A_{T,n}J^i_{T,n}H^i_{T,n}-\Phi_T+v_{T,n}-v^{'}_{T,n}=0,\quad\forall n$$

1. Due to the CRRA utility function (so marginal utility of consumption is positive and Inada condition holds), $\sum_{n=1}^{N}q^i_{T,n}<1$ and $\Phi_t=0$ due to complementary slackness.
2. From the previous result $\sum_{n=1}^{N}q^i_{T,n}<1$, it implies $q^i_{T,n}<1$ and $v^{'}_{T,n}=0$.
3. Due to the CRRA utility function (so marginal utility of consumption is positive), $\mu_T>0$ and $\sum_{n=1}^{N}(J^i_{T,n})^{\sigma}=1$.
4. From $[q^i_{T,n}]$, $v_{T,n}>0$ and it implies $q^i_{T,n}=0$.
5. If $\phi_{T,n}>0$, $J^i_{T,n}=0$, it implies $\phi^{'}_{T,n}=0$ for skill n but not other skills $n^{'}\neq n$. Thus, from $[J_n]$,

$$\underbrace{\frac{(1-\sum_{n=1}^{N}q^i_{T,n})A_{T,n}H^i_{T,n}}{\left[(1-\sum_{n=1}^{N}q^i_{T,n})\sum_{n=1}^{N}A_{T,n}J^i_{T,n}H^i_{T,n}\right]^{1-\eta}}+\phi_{T,n}}_{>0}=\underbrace{\mu_t\sigma J^{i\ \sigma-1}_{T,n}}_{=0},$$

   a contradiction. Thus, $J^i_{T,n}>0$ and $\phi_{T,n}=0\ \forall n$.
6. If $\phi^{'}_{T,n}>0$ so $J^i_{T,n}=1$, it implies $J^i_{T,n'}=0\ \forall n'\neq n$, which contradicts the previous finding [5]. Thus,$J^i_{T,n}<1$ and $\phi^{'}_{T,n}=0$.

All in all, the optimum for occupational choices are interior $J^i_{T,n}=\frac{(A_{T,n}H^i_{T,n})^{\frac{1}{\sigma-1}}}{\left[\sum_{n=1}^{N}(A_{T,n}H^i_{T,n})^{\frac{\sigma}{\sigma-1}}\right]^{\frac{1}{\sigma}}}$ and $q^i_{T,n}=0,\forall n$.

Then, I consider $t<T$, the bellman equation is

$$V_t(\boldsymbol{A_t},\boldsymbol{H^i_t},\boldsymbol{\hat{\theta}^i_t})=\max_{\{\boldsymbol{q^i_t},\boldsymbol{J^i}\}}\frac{\left[(1-\sum_{n=1}^{N}q^i_{t,n})\sum_{n=1}^{N}A_{t,n}J_nH^i_{t,n}\right]^{\eta}-1}{\eta}+\beta\,\mathbb{E}_{\boldsymbol{\epsilon^i_t},\boldsymbol{\gamma_t}}V_{t+1}(\boldsymbol{A_{t+1}},\boldsymbol{H^i_{t+1}},\boldsymbol{\hat{\theta}^i_{t+1}})$$

$$s.t.\quad \text{Eq. (3), (7), (8), (9), (10)}$$

$$\boldsymbol{A_t},\boldsymbol{H^i_t},\boldsymbol{\hat{\theta}^i_t}\text{ are given}$$

The lagrangian is

$$\mathcal{L} = \frac{\left[(1-\sum_{n=1}^{N} q_{t,n}^i)\sum_{n=1}^{N} A_{t,n}J_n H_{t,n}^i\right]^{\eta} - 1}{\eta} + \beta \mathop{\mathbb{E}_t}_{\boldsymbol{\epsilon_t^i},\boldsymbol{\gamma_t}} V_{t+1}(\boldsymbol{A_{t+1}}, \boldsymbol{H_{t+1}^i}, \boldsymbol{\hat{\theta}_{t+1}^i})$$
$$-\sum_{n=1}^{N} \xi_{t,n}\left[H_{t+1,n}^i - (1-\delta)H_{t,n}^i - (\theta_n^i + \epsilon_{t,n}^i)(q_{t,n}^i H_{t,n}^i)^{\alpha}\right] - \mu_t(\sum_{n=1}^{N} J_n^{\sigma} - 1) - \Phi_t(\sum_{n=1}^{N} q_{t,n}^i - 1)$$
$$+\sum_{n=1}^{N}\left[\phi_{t,n}J_n + v_{t,n}q_{t,n}^i - \phi_{t,n}^{'}(J_n - 1) - v_{t,n}^{'}(q_{t,n}^i - 1)\right]$$

The first-order conditions are

$$[J_n]: \left[(1-\sum_{n=1}^{N} q_{t,n}^i)\sum_{n=1}^{N} A_{t,n}J_{t,n}^i H_{t,n}^i\right]^{\eta-1}(1-\sum_{n=1}^{N} q_{t,n}^i)A_{t,n}H_{t,n}^i - \mu_t\sigma(J_{t,n}^i)^{\sigma-1} + \phi_{t,n} - \phi_{t,n}^{'} = 0, \quad \forall n$$

$$[q_{t,n}^i]: -\left[(1-\sum_{n=1}^{N} q_{t,n}^i)\sum_{n=1}^{N} A_{t,n}J_{t,n}^i H_{t,n}^i\right]^{\eta-1}\sum_{n=1}^{N} A_{t,n}J_{t,n}^i H_{t,n}^i + \beta \mathop{\mathbb{E}_t}_{\boldsymbol{\epsilon_t^i},\boldsymbol{\gamma_t}} \frac{\partial V_{t+1}(\boldsymbol{A_{t+1}}, \boldsymbol{H_{t+1}^i}, \boldsymbol{\hat{\theta}_{t+1}^i})}{\partial H_{t+1,n}^i}\frac{\partial H_{t+1,n}^i}{\partial q_{t,n}^i}$$
$$-\Phi_t + v_{t,n} - v_{t,n}^{'} = 0, \quad \forall n$$

where

$$\frac{\partial H_{t+1,n}^i}{\partial q_{t,n}^i} = \alpha(\theta_n^i + \epsilon_{t,n}^i){q_{t,n}^i}^{\alpha-1}{H_{t,n}^i}^{\alpha},$$

$$\frac{\partial V_t(\boldsymbol{A_t}, \boldsymbol{H_t^i}, \boldsymbol{\hat{\theta}_t^i})}{\partial H_{t,n}^i} = \left[(1-\sum_{n=1}^{N} q_{t,n}^i)\sum_{n=1}^{N} A_{t,n}J_{t,n}^i H_{t,n}^i\right]^{\eta-1}(1-\sum_{n=1}^{N} q_{t,n}^i)A_{t,n}J_{t,n}^i$$

I follow the same procedure to analyze.

1. Due to the CRRA utility function (so marginal utility of consumption is positive and Inada condition holds), $\sum_{n=1}^{N} q_{t,n}^i < 1$ and $\Phi_t = 0$ due to complementary slackness.

2. From the previous result $\sum_{n=1}^{N} q_{t,n}^i < 1$, it implies $q_{t,n}^i < 1$ and $v_{t,n}^{'} = 0$.

3. Due to the CRRA utility function (so marginal utility of consumption is positive), $\mu_t > 0$ and $\sum_{n=1}^{N}(J_{t,n}^i)^{\sigma} = 1$.

4. If $v_{t,n} > 0$ which implies $q^i_{t,n} = 0$, from $[q^i_{t,n}]$, I have a contradiction

$$\underbrace{\left[(1-\sum_{n=1}^{N} q^i_{t,n})\sum_{n=1}^{N} A_{t,n}J^i_{t,n}H^i_{t,n}\right]^{\eta-1}\sum_{n=1}^{N} A_{t,n}J^i_{t,n}H^i_{t,n}}_{>0} + \underbrace{v_{t,n}}_{>0}$$
$$= \underbrace{\alpha\beta {q^i_{t,n}}^{\alpha-1} {H^i_{t,n}}^{\alpha} \boldsymbol{\epsilon^i_t, \gamma_t}\mathbb{E}_t\left[(1-\sum_{n=1}^{N} q^i_{t,n})\sum_{n=1}^{N} A_{t,n}J^i_{t,n}H^i_{t,n}\right]^{\eta-1}(1-\sum_{n=1}^{N} q^i_{t,n})A_{t,n}J^i_{t,n}(\theta^i_n+\epsilon^i_{t,n})}_{\to\infty}$$

   Thus, $q^i_{t,n} > 0, v_{t,n} = 0$.

5. If $\phi_{t,n} > 0$, $J^i_{t,n} = 0$, it implies $\phi'_{t,n} = 0$ for skill n but not other skills $n' \neq n$. Thus, from $[J_n]$,

$$\underbrace{\frac{(1-\sum_{n=1}^{N} q^i_{t,n})A_{t,n}H^i_{t,n}}{\left[(1-\sum_{n=1}^{N} q^i_{t,n})\sum_{n=1}^{N} A_{t,n}J^i_{t,n}H^i_{t,n}\right]^{1-\eta}}}_{>0} + \phi_{t,n} = \underbrace{\mu_t\sigma {J^i_{t,n}}^{\sigma-1}}_{=0},$$

   a contradiction. Thus, $J^i_{t,n} > 0$ and $\phi_{t,n} = 0\ \forall n$.

6. If $\phi'_{t,n} > 0$ so $J^i_{t,n} = 1$, it implies $J^i_{t,n'} = 0\ \forall n' \neq n$, which contradicts the previous finding [5]. Thus,$J^i_{t,n} < 1$ and $\phi'_{t,n} = 0$.

Thus, the optimum for occupational choices are interior $J^i_{t,n} = \frac{(A_{t,n}H^i_{t,n})^{\frac{1}{\sigma-1}}}{\left[\sum_{n=1}^{N}(A_{t,n}H^i_{t,n})^{\frac{\sigma}{\sigma-1}}\right]^{\frac{1}{\sigma}}}$ and $0 < q^i_{t,n} < 1, \forall n$.