# The macroeconomic implication of accelerated depreciation

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**Abstract**

Governments widely employ investment subsidy policies to facilitate investment in recessions. This paper examines the efficacy of two investment subsidy policies in the United States in the form of accelerated depreciation, namely, bonus depreciation and maximum allowance. A heterogeneous firm model with both real and financial frictions has been proposed to evaluate the effect of both policies on firm dynamics. The maximum allowance defined by Section 179 is targeted at firms that invest below a threshold. The results show that such policy distorts the investment incentive as firms are willing to sacrifice capital accumulation in exchange for immediate tax deduction by small investment. On the other hand, bonus depreciation applies to all firms without distortion, and thus can increase the overall investment, and its effects are more significant among small firms.

**Keywords:** Tax shields, Collateral constraint, Business cycle, Firm dynamics

**JEL Codes:** E22, E32, E62, H25

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# 1 Introduction

How tax benefit shapes the investment incentive has long been the interests among Economists and policy makers (Hall and Jorgenson (1967)). Accelerated depreciation is one of the most widely used subsidy policies to boost investment, adopted by over 41 countries as of 2018 (Steinmüller et al. (2018)). Despite its popularity, quantitative analyses on evaluating the macroeconomic implication of accelerated depreciation are scarce. In the United States, two distinct forms of accelerated depreciation coexist: bonus depreciation and maximum allowance. When firms undertake investment, they can file corporate tax deductions based on the depreciation schedule during the duration of useful life. The bonus depreciation offers all firm a fraction of the net present value of the total tax deduction at the year of investing, essentially accelerating the schedule. Maximum allowance, on the other hand, is a targeted policy. It grants firms investing below the threshold the entire tax deduction in the year of investment. This dual system of targeted and untargeted policies presents an interesting twist. As the overall bonus rate increases, the relative advantage enjoyed by firms eligible for the maximum allowance shrinks (Ohrn (2019)). As a result, the net effect of combined policies on aggregates become ambiguous.

In this paper, I provide a general equilibrium analysis of both investment subsidy policies through the lens of a heterogeneous firm model with real and financial frictions. The model incorporates collateral constraints, which limit firms' borrowing capacity and introduce financial friction. Ironically, the maximum allowance intended to boost investment from small yet productive firms creates unintended real friction. The threshold and associated tax deductions distort investment incentives for small and young firms. They prioritize immediate tax benefits by investing below the threshold, even though this constrains their capital accumulation and tightened the collateral constraints. Consequently, firms face a critical binary choice each period: sacrifice long-term growth by investing below the threshold for immediate tax relief, or invest heavily above the threshold to build capital, loosen borrowing constraints, and potentially propel their growth trajectory. In contrast, bonus depreciation avoids this trade-off entirely by applying equally to all firms. It awards additional tax benefit to firms investing above the threshold without altering investment incentive to firms investing below the threshold.

The remainder of the paper proceeds as follows. Section 2 discusses the related literature. Section 3 presents a my main theoretical results in stylized model of investment subsidy policies. Section 4 introduces the quantitative model, and Section 5 provides useful analysis for numerical calculation. Section 6 exhibits and discusses the calibration strategies and quantitative results to unconstrained firms, and Section 7 concludes.

# 2 Related literature

There is a large literature investigating how tax incentive influence the aggregate investment. Hall and Jorgenson (1967) is the first to evaluate response of a representative firm to tax credit through the change in the user cost of capital. My work follows this tradition in that the user costs associate to firms' binary investment choice are different. Firms that invest below the threshold enjoys lower user cost than those investing above the threshold, and the difference between two user costs are determined by the rate of bonus depreciation. Furthermore, Summers, Bosworth, Tobin and White (1981) proposed the tax-adjusted Tobin's $q$ to evaluate how tax policies alter the valuation of the firms, providing another channel for fiscal policy to affect capital accumulation. In my model, both the corporate tax and the investment subsidy policies enter the value of the firms, allowing me to examine the Tobin's $q$ channel of tax credit. By unifying both channels in my model, I can identify the source heterogeneous response to tax credits and to evaluate the corresponding aggregate implication.

Earlier empirical literature starts with data on public companies but oftentimes concludes that investment is not responsive to tax credit. Goolsbee (1998) uses data on the prices of capital by Bureau of Economic Analysis (BEA) and concludes that the effect of investment tax credit is offset by the increase in capital prices among public firms. Cummins, Hassett and Hubbard (1996) utilizes panel data among 14 OECD countries and identified that the user cost of capital and the adjustment costs can explain such unresponsiveness. House and Shapiro (2008) matches the BEA data with Internal Revenue Service (IRS) depreciation schedules and analyzes the 2001 to 2002 bonus depreciation. They claim that the intertemporal elasticity of investment is high under two assumptions: capital is long-lived and investment tax credit is temporary and unexpected. Even though the conclusion by House and Shapiro (2008) highlights the importance of intertemporal substitution, they assume all tax responses are temporary price effects and not income effects, which contradicts the evidence documented in corporate finance literature (e.g., Lamont (1997)). The reason why all studies above cannot find the heterogeneity in tax-term elasticity is that they only utilize data on public firms.

Recent empirical literature has utilized firm-level data and state-level policy compliance and found out substantial heterogeneity in investment response. Zwick and Mahon (2017) is the first empirical research that exploits business tax data from IRS to estimate the heterogeneity of investment response to tax credit. They examine the impact of bonus depreciation by comparing industries that use long duration of capital to industries that use short duration. They found that bonus depreciation increases the investment of eligible capital by 10 to 16 percent compared to ineligible capital. Also, small firms respond 95 percent more than big firms. Ohrn (2018) further investigates the effect of corporate tax deductions and concludes that the investment raises by 4.7 percent for those states that complies with federal policies. In a subsequent study, Ohrn (2019) identifies potential conflicts between bonus depreciation and maximum allowance as mentioned before. These heterogeneous response from small and young firms are the calibration targets for my model.

Theoretical exploration on response to fiscal policy has been studied through the representative firms models. Fernández-Villaverde (2010) build a dynamic stochastic general equilibrium (DSGE) model with representative firm and financial constraints to analyze the response to fiscal shocks. Occhino (2022) analyzes the aggregate effects of tax cuts and jobs act without dealing with the heterogeneous response to tax credit nor explore the distortion created by maximum allowance. Later, Occhino (2023) evaluates the effect of corporate tax cuts with accelerated depreciation and assumes the bonus depreciation rate is an AR(1) process. This assumption ignores the countercyclical nature of these policies, and may subject to under-estimation of the policy reaction. My contribution is to bring heterogeneity into the theoretical exploration and quantitatively evaluate the efficacy of these policies.

# 3 Stylized Model

## 3.1 Model Environment

Household is infinitely-lived, risk-neutral, and owns all firms. Household's preference is

$$\sum_{t=0}^{\infty} \beta^t c_t,$$

where $\beta \in (0,1)$ is the discount factor and $c_t$ is consumption. Household lends bonds to the firms, and thus the bond price is $\beta$.

Firms are overlapping generations and live for two periods. Firms are born with zero debt and capital endowment $k$ draw from distribution $G[\underline{k}, \bar{k}]$. Firms produce in both periods and undertake capital and bond decisions given corporate tax rate $\tau^c$, investment credit $\tau^I$, maximum allowance threshold $\bar{I}$, rate of depreciation deduction $\xi$, and bond price $\beta$.

A firm chooses between investments below maximum allowance threshold or above,

$$V = \max\left\{V^i, V^a\right\},$$

where $V^i$ denotes the lifetime value for invest above maximum allowance $\bar{I}$, and $V^a$ represents investment below maximum allowance. I will call $V^i$ as $i$-type firms and $V^a$ as $a$-type firms afterward.

In either case, a firm maximizes its discounted lifetime dividend

$$\max_{D_0, D_1, k', b'} D_0 + \beta D_1$$

subject to

$$\begin{aligned}
D_0 &= f(k) + \beta b' - k' + (1-\delta)k - \tau^c \mathcal{I} \\
D_1 &= f(k') - b' + (1-\delta)k' - \tau^c \mathcal{I}' \\
b' &\leq \theta k' \\
D_0 &\geq 0 \\
D_1 &\geq 0 \\
k' &\geq 0 \\
T' &= (1-\xi)\tau^I(k' - (1-\delta)k) \\
\mathcal{I} &= f(k) - \xi\tau^I(k' - (1-\delta)k) \\
\mathcal{I}' &= f(k') + \tau^I(1-\delta)k' - \delta^T T'
\end{aligned}$$

where $\mathcal{I}$ is the taxable income, and $\xi = 1$ for $a$-type firms. I substitute $\mathcal{I}$ and $T'$ into budget constraints and get

$$\begin{aligned} D_0 &= (1-\tau^c)f(k) + \beta b' - (1-\tau^c\tau^I\xi)(k' - (1-\delta)k) \\ D_1 &= (1-\tau^c)f(k') - b' + (1-\tau^c\tau^I)(1-\delta)k' + \tau^c\delta^T(1-\xi)\tau^I(k'-(1-\delta)k) \end{aligned},$$

Thus, the original problem can be rewritten as

$$\begin{aligned} V^i = \max_{k'>(1-\delta)k+\bar{I},b'} &(1-\tau^c)f(k) + \beta b' - (1-\tau^c\tau^I\xi)(k'-(1-\delta)k) + \beta\Big[(1-\tau^c)f(k') - b' \\ &+ (1-\tau^c\tau^I)(1-\delta)k' + \tau^c\delta^T(1-\xi)\tau^I(k'-(1-\delta)k)\Big] + \lambda(\theta k' - b'), \\ V^a = \max_{k'\le(1-\delta)k+\bar{I},b'} &(1-\tau^c)f(k) + \beta b' - (1-\tau^c\tau^I)(k'-(1-\delta)k) + \beta\Big[(1-\tau^c)f(k') - b' \\ &+ (1-\tau^c\tau^I)(1-\delta)k'\Big] + \lambda(\theta k' - b'), \end{aligned}$$

where $\lambda$ is the Lagrangian multiplier for collateral constraint.

The optimal demand for capital satisfies the following first-order conditions

$$(1-\tau^c\tau^I\xi) = \beta(1-\tau^c)f_k(k') + \beta(1-\tau^c\tau^I)(1-\delta) + \beta\tau^c\tau^I\delta^T(1-\xi) + \lambda\theta, \quad (1)$$

$$(1-\tau^c\tau^I) = \beta(1-\tau^c)f_k(k') + \beta(1-\tau^c\tau^I)(1-\delta) + \lambda\theta. \quad (2)$$

Let $c^i$ and $c^a$ denote the user cost of capital of $i$-type and $a$-type firms following Jorgenson (1963) and Hall and Jorgenson (1967). I rearrange equation (1) and get

$$c^i \equiv \frac{1-\tau^c\tau^I\xi}{1-\tau^c} - \beta(1-\delta)\frac{1-\tau^c\tau^I}{1-\tau^c} - \beta\frac{\tau^c\tau^I}{1-\tau^c}\delta^T(1-\xi) = \beta f_k(k') + \frac{\lambda\theta}{1-\tau^c},$$

The first term, $\frac{1-\tau^c\tau^I\xi}{1-\tau^c}$, denotes the after-tax down payment per unit of capital at date 0. The second term, $\beta(1-\delta)\frac{1-\tau^c\tau^I}{1-\tau^c}$, represents the after-tax resale value of capital at date 1. The third term, $\beta\frac{\tau^c\tau^I}{1-\tau^c}\delta^T(1-\xi)$, denotes the date 1 tax deduction from the date 0 investment. As the bonus rate $\xi$ increases, both the down payment at date 0 and the tax deduction at date 1 shrink. However, the down payment decreases at a faster speed, leading to a lower user cost as the bonus rate increases. For $a$-type firms, the user cost $c^a$ can be derived from equation (2)

$$c^a \equiv \frac{1-\tau^c\tau^I}{1-\tau^c}(1-\beta(1-\delta)) = \beta f_k(k') + \lambda\theta.$$

Both the after-tax down payment and resale value per unit of capital are the same before discounting and depreciation, and there is no lingering tax deduction. However, this only applies to firms who invest below the maximum allowance $\bar{I}$. Therefore, such a policy is going to induce firms to undertake small investments. They enjoy the tax benefit immediately and are reluctant to make large investments.

Let $f(k) = k^\alpha$, the $i$-type firms' unconstrained target capital is

$$k_i^* = \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}} \left(\frac{1}{\beta}\frac{1-\tau^c\tau^I\xi}{1-\tau^c} - \frac{1-\tau^c\tau^I}{1-\tau^c}(1-\delta) - \frac{\tau^c\tau^I}{1-\tau^c}\delta^T(1-\xi)\right)^{\frac{1}{\alpha-1}}.$$

The $i$-type capital decision $k_i$ is to set its policy as close to $k_i^*$ as maximum allowance permits, i.e., $k_i = \max\left\{(1-\delta)k + \bar{I}, k_i^*\right\}$. Similarly, $a$-type firms' unconstrained target capital is

$$k_a^* = \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}} \left(\left(\frac{1}{\beta} - (1-\delta)\right)\frac{1-\tau^c\tau^I}{1-\tau^c}\right)^{\frac{1}{\alpha-1}},$$

and the corresponding capital decision rule is $k_a = \min\left\{(1-\delta)k + \bar{I}, k_a^*\right\}$. It is clear that $k_a^* > k_i^*$, as the user cost $c_a < c_i$.

When firm is constrained, I set $b' = \theta k'$ and solve the capital choice by zero dividend policy,

$$\bar{k}_i = \frac{(1-\tau^c)f(k) + (1-\tau^c\tau^I\xi)(1-\delta)k}{1-\tau^c\tau^I\xi - \beta\theta}$$
$$\bar{k}_a = \frac{(1-\tau^c)f(k) + (1-\tau^c\tau^I)(1-\delta)k}{1-\tau^c\tau^I - \beta\theta}$$

## 3.2 Parameterization and Results

Table 1 lists the parameters used in the stylized model. Most of the parameters are consistent with the quantitative model. The left panel in figure 1 shows the capital decision rules $k'$ given the initial capital endowment $k$. I assume the initial capital endowment $k$ is uniformly distributed between $[0.05, 4.0]$. The target capital for $a$-type firms are 2.909, represented by the flat line on the right-hand side of the $(1-\delta)k + \bar{I}$ line, and that for $i$-type firms is 2.273, shown in the flat section on the left-hand side. Any $k'$ choice that is above the $(1-\delta)k$ line indicates positive investment. Among all investing firms, the majority of small firms with $k < 0.5$, and median-sized firms are investing exactly at maximum allowance $\bar{I}$. This property qualitatively matches the empirical pattern in Zwick and Mahon (2017), where firms are bunching around the maximum allowance in investment distribution.

Firms are categorized in two dimensions: their financial position and their binary investment decision. Their financial position is represented by a line labeled firm type in figure 1, as defined in Jo and Senga (2019). Type 0 firms are financially unconstrained, and their capital investment is not affected by their bond decision. Type 1 firms have to pay negative date 0 dividend $D_0$ if they do not borrow. They borrow $\beta b'$ to satisfy (1) paying at least zero dividend, and (2) undertaking $k_i^*$ or $k_a^*$ based on their investment decision. Type 2 firms, on the other hand, cannot achieve either $k_i^*$ or $k_a^*$ as their collateral constraint is binding. Their capital decision is distorted to either $\bar{k}_i$ or $\bar{k}_a$ to ensure non-negative dividend payment. The green line (binary choice) represents a binary investment decision between $i$-type and $a$-type. Number 1 is for $a$-type firms, and $i$-type firms are number 0.

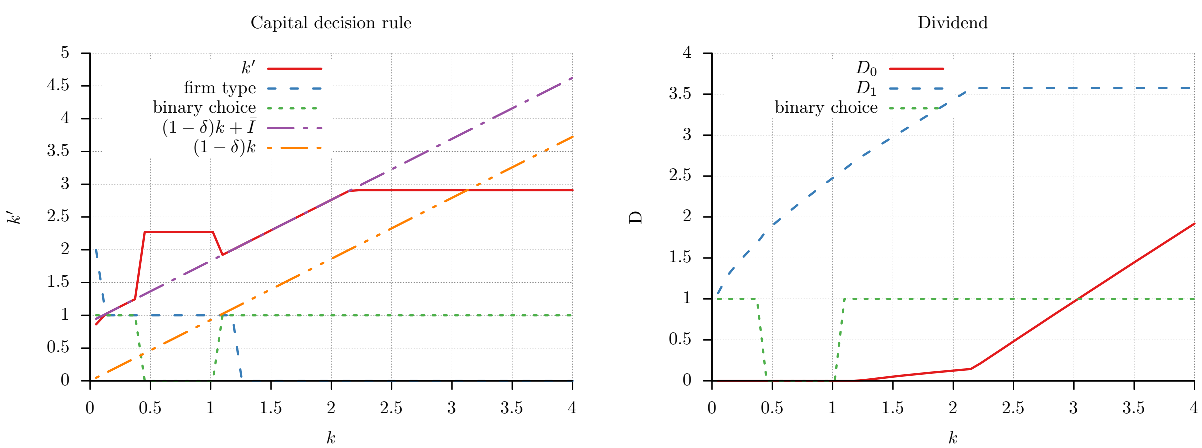
Firms' reactions to both investment subsidy policies depend on their initial capital endowment. All unconstrained firms are $a$-type firms. As they are endowed with high initial capital, they invest below $\bar{I}$ to enjoy less user cost of capital. Also, as both target capital, $k_i^*$ and $k_a^*$, do not depend on maximum allowance $\bar{I}$, firms that stay in action ($k' = (1-\delta)k$) when the maximum allowance is zero are now undertaking positive investment up to $\bar{I}$. Constrained firms, on the contrary, invest larger than $\bar{I}$ and become $i$-type to alleviate their collateral constraint. As collateral constraints are forward-looking, the benefit from looser borrowing limit outweighs the higher user cost that $i$-type firms bear. As firms become more constrained, the benefit from credit is dominated by the user cost, and thus firms choose to be $a$-type when their capital endowment is too low.

The efficacy of both policy tools can be seen in figure 2. In the left panel, I conduct the comparative statics on the effect of bonus depreciation rate $\xi$, holding all other values as specified in table 1. As the binary choice depends on the bonus rate, as I increase the level of $\xi$, the gap between two capital targets decreases, and eventually both targets become the same as $\xi = 1$. As the real friction has been eliminated, all firms who originally invested at level $k_i^*$ are now investing at the same level as $k_a^*$. In the right panel, similar comparative statics have been done, and the increment of maximum allowance $\bar{I}$ may backfire on its goal to help small firms. For those unconstrained firms that are newly eligible to 100% depreciation deduction, their capital stock increases from $k_i^*$ to $(1-\delta)k + \bar{I}$. However, such policy induces those unconstrained firms that are right below the threshold to undertake optimal investment, as they are not willing to pay the higher user cost corresponding to $i$-type investment. As a result, the aggregate effect of the maximum allowance depends on the shape of firm distribution. I will discuss this further in the quantitative model. Furthermore, the higher maximum allowance will distort constrained firms' investment motives and defeat the policy goal. Constrained firms that will invest up to $\bar{k}_i$ when $\bar{I} = 0$ merely invest $(1-\delta)k + \bar{I}$ to be eligible for all tax deduction on date 0. Such distortion is alleviated when almost all firms are eligible to $a$-type investment, i.e., when $\bar{I} = 2$.

Table 1: Parameters for stylized model

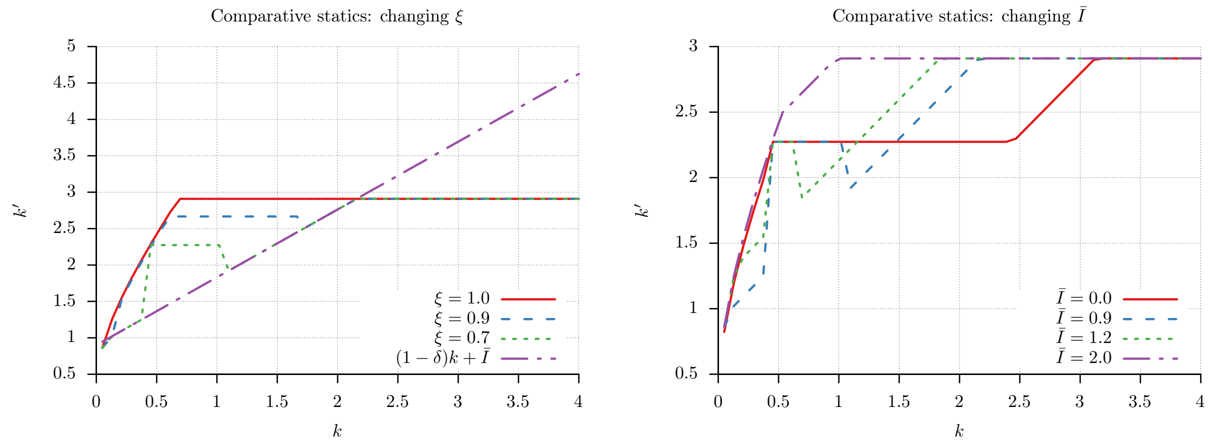
| | Parameter | Value | Reason |
|---|---|---|---|
| Discount rate | $\beta$ | 0.96 | 4% real interest rate |
| Curvature of production function | $\alpha$ | 0.6 | private capital-output ratio |
| Collateralizability | $\theta$ | 0.5 | Li, Whited and Wu (2012) |
| Corporate tax rate | $\tau^c$ | 0.21 | US Tax schedule after TCJA |
| Investment tax benefit | $\tau^I$ | 0.35 | |
| Capital depreciation rate | $\delta$ | 0.069 | average investment-capital ratio |
| Tax benefit depreciation rate | $\delta^T$ | 0.138 | $\delta^T = 2\delta$ |
| Bonus depreciation rate in baseline | $\xi$ | 0.7 | policy tools |
| Maximum allowance | $\bar{I}$ | 0.9 | policy tools |

Figure 1: Capital decision, dividend payment, and firm type




Firm types follow the definition in Jo and Senga (2019): type 0 firms are financially unconstrained, type 1 firms can adopt unconstrained target capital, but have to borrow debt, type 2 firms borrow up to collateral value and undertake $\bar{k}_i$. binary choice are either investing smaller than $\bar{I}$ (number 1) or larger than $\bar{I}$ (number 0).

Figure 2: Comparative statics: changing $\xi$ and $\bar{I}$




# 4 Quantitative Model

Time is discrete and infinite. In my model economy, firms face corporate taxes and collateral constraints. Corporate taxes can be deducted from interest payments and investment expenses. The heterogeneity in the deduction on investment expense defined by the maximum allowance in Section 179 arbitrarily creates real friction by targeting firms that invest small amounts. I begin by describing the model economy. I introduce the optimization problem that firms face, followed by discussing the representative household problem and defining recursive equilibrium.

## 4.1 Firms

There is a continuum of heterogeneous firms, each producing homogeneous output using predetermined capital stock $k$ and labor $n$. The production function is $y = z\varepsilon F(k,n)$, where $F(k,n)$ is an increasing and concave function. The variable $z$ denotes the exogenous TFP shocks that are common among firms, while $\varepsilon$ is a firm-specific stochastic shock. I assume that $\varepsilon$ is a Markov chain, i.e., $\varepsilon \in \mathbf{E} \equiv \{\varepsilon_1, \ldots \varepsilon_{N_\varepsilon}\}$, where $\Pr(\varepsilon' = \varepsilon_j | \varepsilon = \varepsilon_i) = \pi^{\varepsilon}_{ij}$, and $\sum_{j=1}^{N_\varepsilon} \pi^{\varepsilon}_{ij} = 1$. Following Khan and Thomas (2013), I assume each firm faces exogenous exit shock $\pi_d \in (0,1)$ to prevent all firms from accumulating sufficient resources, and none are financially constrained.

At the beginning of each period, a firm is defined by four states: (1) its predetermined capital stock $k \in \mathbf{K} \subset \mathbb{R}_+$, (2) its level of one-period debt $b \in \mathbf{B} \subset \mathbb{R}$ issued one period ahead, (3) its realized idiosyncratic productivity $\varepsilon \in \mathbf{E}$, and (4) its unrealized tax deduction from investment $T \in \mathbf{T} \subset \mathbb{R}_+$. The distribution of firms $\mu$ over $(k, b, T, \varepsilon)$ is defined on a Borel algebra $\mathcal{S} = \mathbf{K} \times \mathbf{B} \times \mathbf{T} \times \mathbf{E}$. Given all individual states, the firm maximizes the expected discounted value function by choosing current employment level $n$, future capital stock $k'$, and next-period debt level $b'$. For each unit of labor employed, the firm pays competitive wage $w$, which depends on the distribution of the firms. The firm can issue one-period debt at a risk-free price $q$ but subject to collateral constraint. The amount of newly-issued debt, $b'$, shall not exceed $\theta$ fraction firm's future capital choice $k'$, i.e., $b' \leq \theta k'$. If the fraction $\theta$ is close to the risk-free interest rate is $\frac{1}{q}$, then the financial constraints become looser. This assumption is based on the limited enforceability of financial contracts. The forward-looking nature of collateral constraints follows the specification as in Kiyotaki and Moore (1997). Furthermore, the firm can deduct $\tau^b$ fraction of its interest payment $b$ from the taxable income $\mathcal{I}$, which will be defined later.

The firm's capital decision, $k'$, has two impacts on its value: production and tax deduction. The investment $I$ is determined by the standard accumulation equation, $I = k' - (1-\delta)k$, where $\delta \in (0,1)$ is the depreciation rate of capital. For each unit of investment, a firm undertakes, it gets $\tau^I I$ unit of deduction on taxable income throughout the depreciation schedule defined by IRS. If the firm invests below or equal to the maximum allowance $\bar{I}$, i.e., $I \leq \bar{I}$, it gets the entirety of $\tau^I I$ deduction on taxable income today. On the other hand, if the firm invests more than the $\bar{I}$, it only gets $\xi$ fraction of $\tau^I I$ tax deduction. All of the remaining tax deduction $(1-\xi)\tau^I I$ is added to the state $T'$ following the law of motion

$$T' = (1-\delta^T)T + (1-\xi)\tau^I I,$$

where $\delta^T$ is the depreciation rate for the remaining tax benefit, and each period $\delta^T T$ of tax deduction will be realized. I assume $\delta^T > \delta$ to show the accelerated depreciation allowed by the government. For dis-investing firms, i.e., $I < 0$, their taxable income is going to increase due to the capital gain, and $\tau^I$ becomes the tax rate of capital gain.

The government taxes the firms through corporate tax and pays government spending $G$. The taxable income, $\mathcal{I}$, is defined as

$$\mathcal{I} = z\varepsilon F(k,n) - wn - \tau^b b - (\mathcal{J}(I)\tau^I I + \delta^T T),$$

where $\mathcal{J}(I)$ is the indicator function with respect to maximum allowance $\bar{I}$. $\mathcal{J}(I) = 1$ if $I \leq \bar{I}$; $\mathcal{J}(I) = \xi$ if $I > \bar{I}$. The deduction from interest payment, $\tau^b b$, and from capital depreciation, $\mathcal{J}(I)\tau^I I + \delta^T T$, alter the effective tax rate per unit of capital invested. The firm's budget constraint under corporate tax is defined as

$$D = z\varepsilon F(k,n) - wn - b + qb' - (k' - (1-\delta)k) - \tau^c \mathcal{I},$$

where $\tau^c$ is the corporate tax rate, and $D$ is the dividend payment. I combine the common terms and rewrite the budget constraint as

$$D = (1-\tau^c)(z\varepsilon F(k,n)-wn)-(1-\tau^c\tau^b)b+qb'-(1-\tau^c\mathcal{J}(k'-(1-\delta)k)\tau^I)(k'-(1-\delta)k)+\tau^c\delta^T T.$$

Notice that all investment subsidies are in the form of corporate tax deductions. Therefore, if the government decreases the corporate tax $\tau^c$ to zero, the model falls back to the ordinary business cycle model.

I now start to illustrate the problem solved by each firm in the model. Let $v^0(k, b, T, \varepsilon; \mu)$ denote the expected discounted value of a firm at the beginning of the period before the realization of the exogenous exit shock $\pi_d$. Upon exiting, the firm chooses labor demand $n$, sells capital, and repays debts. The function equations are defined as

$$\begin{aligned} v^0(k, b, T, \varepsilon; \mu) = \pi_d \max_n \Big\{ &(1-\tau^c)(z\varepsilon F(k,n) - wn) - (1-\tau^c\tau^b)b \\ &+ qb' + (1-\tau^c\tau^I)(1-\delta)k + \tau^c\delta^T T \Big\} \\ &+ (1-\pi_d)v(k, b, T, \varepsilon; \mu) \end{aligned} \tag{3}$$

Conditional on survival, the continuation problem is stated as a binary choice between investing less versus larger than the maximum allowance,

$$v(k, b, T, \varepsilon; \mu) = \max \left\{ v^a(k, b, T, \varepsilon; \mu), v^i(k, b, T, \varepsilon; \mu) \right\}, \tag{4}$$

where $v^a(k, b, T, \varepsilon; \mu)$ denotes the value to invest below maximum allowance, and $v^i(k, b, T, \varepsilon; \mu)$ represent the value to invest larger than maximum allowance accordingly.

In both cases, the firm is maximizing the current dividend $D$ and expected discounted future firm value. Let $Q(\mu)$ denote the stochastic discounting factor for firms' next-period value given the distribution $\mu$. The dynamic problem for those firms that undertake investment larger than the maximum allowance is

$$v^i(k, b, T, \varepsilon_i; \mu) = \max_{D,k',b',n} D + Q(\mu) \sum_{j=1}^{N_\varepsilon} \pi^\varepsilon_{ij} v^0(k', b', T', \varepsilon_j; \mu'), \tag{5}$$

subject to

$$\begin{aligned} 0 \leq D = &(1-\tau^c)(z\varepsilon F(k,n) - wn) - (1-\tau^c\tau^b)b \\ &+ qb' - (1-\tau^c\xi\tau^I)(k' - (1-\delta)k) + \tau^c\delta^T T. \end{aligned} \tag{6}$$

$$k' > (1-\delta)k + \bar{I} \tag{7}$$

$$b' \leq \theta k' \tag{8}$$

$$T' = (1 - \delta^T)T + (1 - \xi)\tau^I(k' - (1 - \delta)k) \tag{9}$$

$$\mu' = \Gamma(\mu) \tag{10}$$

The counterpart for firms that undertake investment below the maximum allowance is

$$v^a(k, b, T, \varepsilon_i; \mu) = \max_{D,k',b',n} D + Q(\mu) \sum_{j=1}^{N_\varepsilon} \pi^\varepsilon_{ij} v^0(k', b', T', \varepsilon_j; \mu'), \tag{11}$$

subject to

$$0 \leq D = (1 - \tau^c)(z\varepsilon F(k, n) - wn) - (1 - \tau^c \tau^b)b + qb' - (1 - \tau^c \tau^I)(k' - (1 - \delta)k) + \tau^c \delta^T T. \tag{12}$$

$$k' \leq (1 - \delta)k + \bar{I} \tag{13}$$

$$b' \leq \theta k' \tag{14}$$

$$T' = (1 - \delta^T)T \tag{15}$$

$$\mu' = \Gamma(\mu) \tag{16}$$

Since there is no friction regarding the firm's employment decision, it pays the current wage bill after production, and the future capital decision does not affect current production. Therefore, the employment choice does not depend on the debt choice, remaining tax benefit, or continuation value. Denote the policy functions associate to firm's employment be $N(k, \varepsilon; \mu)$, capital be $K(k, b, T, \varepsilon; \mu)$, debt be $B(k, b, T, \varepsilon; \mu)$, dividend be $D(k, b, T, \varepsilon; \mu)$, and remaining tax benefit be $T(T, K(k, b, T, \varepsilon; \mu), k; \mu)$. I characterize these policy functions in section 5.

## 4.2 Household

Following the specification of Khan and Thomas (2013), I assume there is a unit measure of identical households in the model. In each period, representative households maximize their lifetime utility by choosing consumption, $c$, labor supply, $n^h$, future firm shareholding, $\lambda'$, and future bond holding, $a'$:

$$\begin{aligned} V^h(\lambda, a; \mu) = & \max_{c, n^h, a', \lambda'} \left\{ u(c, 1-n^h) + \beta V^h(\lambda', a'; \mu') \right\} \\ \text{s.t.} \quad & c + qa' + \int \rho_1(k', b', T', \varepsilon')\lambda'(d[k' \times b' \times T' \times \varepsilon']) \\ & \leq w(\mu)n^h + a + \int \rho_0(k, b, T, \varepsilon)\lambda(d[k \times b \times T \times \varepsilon]) \end{aligned} , \tag{17}$$

where $\rho_0(k, b, T, \varepsilon)$ is the dividend-inclusive price of the current share, and $\rho_1(k', b', T', \varepsilon')$ is the ex-dividend price of the future share. Let $C^h(\lambda, a; \mu)$ be the consumption demand function, and $N^h(\lambda, a; \mu)$ is the labor supply function. Similarly, let $A^h(\lambda, a; \mu)$ denote the households' decision for the bond, and $\Lambda(\lambda, a; \mu)$ is the choice of firm shares.

## 4.3 Recursive Equilibrium

A *recursive competitive equilibrium* is a set of functions including prices $(w, q, \rho_0, \rho_1)$, quantities $\left(N, K, B, D, T, C^h, N^h, A^h, \Lambda\right)$, a distribution $\mu(k, b, T, \varepsilon)$, and $\left(v^0, v^a, v^i, v, V^h\right)$ that solve firms' and household's optimization problems and clear the markets for assets, labor, and output in the following conditions.

1. $v^0$, $v^a$, $v^i$, and $v$ solve (3), (4), (5), and (11). The associated policy functions for firms are $(N, K, B, D, T)$.
2. $V^h$ solves (17), and the associated policy functions for households are $(C^h, N^h, A^h, \Lambda)$
3. Labor market clears, i.e., $N^h(\mu, a; \mu) = \int_{\mathcal{S}} N(k, \varepsilon; \mu)\mu(d[k \times b \times T \times \varepsilon])$.
4. Goods market clears, i.e.,

$$\begin{aligned} C^h(\mu, a; \mu) = \int_{\mathcal{S}} \Big\{ & (1-\tau^c)z\varepsilon F(k, N(k, \varepsilon; \mu) - wN(k, \varepsilon; \mu)) \\ & - (1-\pi_d)[1 - \tau^c\tau^I \mathcal{J}(K(k, b, T, \varepsilon; \mu) - (1-\delta)k)] \\ & \times [K(k, b, T, \varepsilon; \mu) - (1-\delta)k] + \tau^c\delta^T T \\ & + \pi_d((1-\tau^c\tau^I)(1-\delta)k - k_0) \Big\} \mu(d[k \times b \times T \times \varepsilon]) - G, \end{aligned}$$

   where $\mathcal{J}(I) = 1$ if $I \leq \bar{I}$, and $\xi$ otherwise. Two variables need to be pinned down: $G$ and $k_0$. $G$ is the exogenous government spending that satisfied the budget balance condition,

$$G = \int_{\mathcal{S}} \Big\{ \tau^c \Big[ z\varepsilon F(k, N(k,\varepsilon;\mu)) - w(\mu)N(k,\varepsilon;\mu) - \tau^b b - \tau^I \mathcal{J}(K(k,b,T,\varepsilon;\mu) - (1-\delta)k) \\ \times (K(k,b,T,\varepsilon;\mu) - (1-\delta)k) - \delta^T T \Big] \Big\} \mu(d[k \times b \times T \times \varepsilon]).$$

$k_0$ is the capital endowment for entrants. I assume $k_0$ is a fixed fraction $\chi$ of the long-run aggregate capital stock,

$$k_0 = \chi \int k \tilde{\mu}(d[k \times b \times T \times \varepsilon]), \tag{18}$$

where $\tilde{\mu}$ is the steady-state distribution.

5. The distribution of firms, $\mu(k, b, T, \varepsilon)$, is a fixed point of $\Gamma$ function. $\Gamma(\mu)$ is consistent with policy functions $(K, B, T)$ and law of motion of $\varepsilon$.

# 5 Analysis

Before solving the recursive competitive equilibrium, I reformulate the firm's problem by exploiting the optimality conditions implied by the household's problem. In equilibrium, the wage $w$ is pinned down by the marginal rate of substitution between consumption and leisure, that is,

$$w(\mu) = \frac{D_2 u(c, 1-n^h)}{D_1 u(c, 1-n^h)}.$$

Similarly, the bond price $q$ equals the inverse of the expected real interest rate. As there is no aggregate uncertainty in the economy, the expected real interest rate is $\frac{1}{\beta}$. The stochastic discounting factor $Q(\mu)$ equals to household's consumption across states,

$$Q(\mu) = \beta \frac{D_1 u(c', 1-n^{h\prime})}{D_1 u(c, 1-n^h)}.$$

Without the loss of generality, I define $p(\mu)$ to be the marginal utility of consumption, $D_1 u(c, 1-n^h)$. The $p(\mu)$ represents the output price that is used to evaluate the firm's current dividend.

After incorporating the household's optimality condition into the prices that firms face, I define a new value $V$ as the multiplication between $p(\mu)$ and $v$, and rewrite dynamic problem (3), (4), (5), and (11):

$$\begin{aligned} V^0(k,b,T,\varepsilon;\mu) = p(\mu)\pi_d \max_n \Big\{ &(1-\tau^c)(z\varepsilon F(k,n) - wn) - (1-\tau^c\tau^b)b \\ &+ qb' + (1-\tau^c\tau^I)(1-\delta)k + \tau^c\delta^T T \Big\} \\ &+ (1-\pi_d)V(k,b,T,\varepsilon;\mu), \end{aligned} \tag{19}$$

where

$$V(k,b,T,\varepsilon;\mu) = \max\left\{V^a(k,b,T,\varepsilon;\mu), V^i(k,b,T,\varepsilon;\mu)\right\}., \tag{20}$$

The dynamic problem for firms who invest larger than the maximum allowance is

$$V^i(k,b,T,\varepsilon_i;\mu) = \max_{D,k',b',n} p(\mu)D + \beta \sum_{j=1}^{N_\varepsilon} \pi^{\varepsilon}_{ij} V^0(k',b',T',\varepsilon_j;\mu'), \tag{21}$$

subject to constraints (6)-(10). The counterpart for firms that undertake investment below the maximum allowance is

$$V^a(k, b, T, \varepsilon_i; \mu) = \max_{D,k',b',n} p(\mu)D + \beta \sum_{j=1}^{N_\varepsilon} \pi_{ij}^\varepsilon V^0(k', b', T', \varepsilon_j; \mu'), \tag{22}$$

subject to constraints (12)-(16).

I start my analysis by deriving the optimal labor choice $N(k, \varepsilon)$. Since there is no friction in the labor market, a firm's labor demand is independent of intertemporal choices. In other words, the optimal labor choice can be derived by solving $\pi(k, \varepsilon) \equiv \max_n z\varepsilon F(k, N(k, \varepsilon)) - wN(k, \varepsilon)$ and get

$$N(k, \varepsilon) = \left(\frac{\nu z \varepsilon k^\alpha}{w}\right)^{\frac{1}{1-\nu}}.$$

Thus, the flow profit $\pi(k, \varepsilon)$ is rewritten as

$$\pi(k, \varepsilon) = A(w) z^{\frac{1}{1-\nu}} \varepsilon^{\frac{1}{1-\nu}} k^{\frac{\alpha}{1-\nu}}, \tag{23}$$

where $A(w) = \left[\left(\frac{\nu}{w}\right)^{\frac{\nu}{1-\nu}} - w\left(\frac{\nu}{w}\right)^{\frac{1}{1-\nu}}\right]$.

To characterize a firm's intertemporal decision, I follow Khan and Thomas (2013) and Jo and Senga (2019) and separate firms into *unconstrained* and *constrained*. Unconstrained firms are those that have already accumulated enough financial savings such that the collateral constraints will never bind in all possible states. Thus, they are indifferent between paying dividends and financial savings. Following Khan and Thomas (2013), I resolve this indeterminacy by requiring unconstrained firms to adapt *minimum saving policy*, i.e., they prioritize dividend payment and accumulate the lowest financial saving $b'$ to stay unconstrained.

Let $W$ function be the value function for unconstrained firms. The start-of-period value before the realization of exit shocks, $W^0$, is

$$\begin{aligned} W^0(k, b, T, \varepsilon; \mu) = p(\mu)\pi_d \Big[&(1 - \tau^c)\pi(k, \varepsilon) - (1 - \tau^c \tau^b)b \\ &+ qb' + (1 - \tau^c \tau^I)(1 - \delta)k + \tau^c \delta^T T\Big] \\ &+ (1 - \pi_d)W(k, b, T, \varepsilon; \mu). \end{aligned}$$

Upon survival, unconstrained firms undertake binary choice similar to (4),

$$W(k, b, T, \varepsilon; \mu) = \max\left\{W^a(k, b, T, \varepsilon; \mu), W^i(k, b, T, \varepsilon; \mu)\right\}.$$

As the capital decision of the unconstrained firm is orthogonal to its bond decision and the firm is indifferent about the bond choice, I express the firm's current value as $W(k,b,T,\varepsilon;\mu) = W(k,0,T,\varepsilon;\mu) - pb$ and the start-of-period value as $W^0(k,b,T,\varepsilon;\mu) = W^0(k,0,T,\varepsilon;\mu) - pb$. Given these transformation, I rewrite (5) and (11) as

$$W^i(k,b,T,\varepsilon_i;\mu) = p\pi(k,\varepsilon) - p(1-\tau^c\tau^b)b + p(1-\tau^c\tau^I\xi)(1-\delta)k + \tau^c\delta^T T \\ + \max_{k'>\bar{I}+(1-\delta)k}\left\{-p(1-\tau^c\tau^I\xi)k' + \beta\sum_{j=1}^{N_\varepsilon}\pi^\varepsilon_{ij}W^0(k',0,T',\varepsilon_j;\mu')\right\},$$

and

$$W^a(k,b,T,\varepsilon_i;\mu) = p\pi(k,\varepsilon) - p(1-\tau^c\tau^b)b + p(1-\tau^c\tau^I)(1-\delta)k + \tau^c\delta^T T \\ + \max_{k'>\bar{I}+(1-\delta)k}\left\{-p(1-\tau^c\tau^I)k' + \beta\sum_{j=1}^{N_\varepsilon}\pi^\varepsilon_{ij}W^0(k',0,T',\varepsilon_j;\mu')\right\},$$

where $\pi(k,\varepsilon)$ is defined by (23).

To search for the target capitals that solve the above two problems, it is necessary to find conditional expected start-of-period value function $W^0(k',0,T',\varepsilon_j;\mu)$. Yet, the future tax benefit $T'$ is a function of current tax benefit $T$ and current capital stock $k$. Therefore, the $i$-type target capital, $k^*_i(k,T,\varepsilon)$, and $a$-type target capital, $k^*_a(k,T,\varepsilon)$, are both functions of $k$ and $T$. To be specific, $k^*_i(k,T,\varepsilon)$ is the target capital that a firm will choose given the after-tax relative price of investment as $1-\tau^c\tau^I\xi$, and correspondingly $k^*_a(k,T,\varepsilon)$ is the target capital given the relative price is $1-\tau^c\tau^I$:

$$k^*_i(k,T,\varepsilon) = \arg\max_{k'>\bar{I}+(1-\delta)k}\left\{-p(1-\tau^c\tau^I\xi)k' + \beta\sum_{j=1}^{N_\varepsilon}\pi^\varepsilon_{ij}W^0(k',0,T',\varepsilon_j;\mu')\right\},$$
$$k^*_a(k,T,\varepsilon) = \arg\max_{k'>\bar{I}+(1-\delta)k}\left\{-p(1-\tau^c\tau^I)k' + \beta\sum_{j=1}^{N_\varepsilon}\pi^\varepsilon_{ij}W^0(k',0,T',\varepsilon_j;\mu')\right\}.$$

Thus, the capital decision rule for unconstrained firms, $K^w(k,T,\varepsilon)$, is

$$K^w(k,T,\varepsilon) = \begin{cases} k^*_i(k,T,\varepsilon) & \text{if } W^i(k,b,T,\varepsilon_i;\mu) > W^a(k,b,T,\varepsilon_i;\mu) \\ k^*_a(k,T,\varepsilon) & \text{if } W^i(k,b,T,\varepsilon_i;\mu) \le W^a(k,b,T,\varepsilon_i;\mu) \end{cases}.$$

As the expected discounted value function $W^0(\cdot)$ is not differentiable, I cannot derive the user cost by first-order conditions. Nevertheless, the user cost of capital for both types of firms should be the same as the two-period model. To elaborate, $a$-type firms enjoy all of the tax benefits immediately, and thus the user cost is the same as in the stylized model,

$$c^a = \frac{1-\tau^c\tau^I}{1-\tau^c}(1-\beta(1-\delta)).$$

On the contrary, the investment tax deduction enjoyed by $i$-type firms occur in the future. Let the firm purchase capital at time 0 and sell capital at time $t$, where the reason for selling can be disinvesting or fire sale due to exit shock. The user cost of capital for the firm is

$$c^i(t) \equiv \frac{1-\tau^c\tau^I\xi}{1-\tau^c} - \sum_{j=1}^{t}\left[\beta^j\left(\delta^T\right)^j(1-\xi)\frac{\tau^c\tau^I}{1-\tau^c}\right] - \beta^t(1-\delta)^t\frac{1-\tau^c\tau^I}{1-\tau^c},$$

where the second and third terms represent the present discounted value of tax deduction and resale value of capital, respectively. If the firm sells the capital at time 1, the user cost is the same as in the stylized model,

$$c^i(1) \equiv \frac{1-\tau^c\tau^I\xi}{1-\tau^c} - \beta\delta^T(1-\xi)\frac{\tau^c\tau^I}{1-\tau^c} - \beta(1-\delta)\frac{1-\tau^c\tau^I}{1-\tau^c}.$$

If the firm never sells this equipment, the third term goes to zero, and thus we can rewrite the user cost as

$$c^i(\infty) \equiv \frac{1-\tau^c\tau^I\xi}{1-\tau^c} - \frac{1-\xi}{1-\beta\delta^T}\frac{\tau^c\tau^I}{1-\tau^c}.$$

Clearly, $c^i(t)$ is an increasing function of $t$ as $c^i(\infty) > c^i(1)$. Thus, it is always better for firms to sell this equipment after one period and only bear small user cost $c^i(1)$, and the user cost of $i$-type firms should be $c^i \equiv c^i(1)$.

The *minimum saving policy*, $B^w(k,T,\varepsilon)$, can be recursively calculated by the following two equations with both policy functions for labor, $N(k,\varepsilon)$, and capital, $K^w(k,T,\varepsilon)$,

$$\begin{aligned}
B^w(k,T,\varepsilon) &= \min_{\varepsilon_j}\left(\tilde{B}(K^w(k,T,\varepsilon_i),T',\varepsilon_j)\right)\\
\tilde{B}(k,T,\varepsilon_i) &= (1-\tau^c)\pi(k,\varepsilon_i) + \tau^c\delta^T T\\
&\qquad - (1-\tau^c\tau^I\mathcal{J}\left(K^w(k,T,\varepsilon_i)-(1-\delta)k\right))\left(K^w(k,T,\varepsilon_i)-(1-\delta)k\right)\\
&\qquad + q\min\left\{B^w(k,T,\varepsilon_i),\theta K^w(k,T,\varepsilon_i)\right\},
\end{aligned}$$

where $\tilde{B}(k,T,\varepsilon)$ represents the minimum level of saving (negative debt) that an unconstrained firm needs to put aside to remain unconstrained given the realization of $\varepsilon_j$. $B^w(k,T,\varepsilon)$, therefore, is the minimum of $\tilde{B}(K^w(\cdot),T',\varepsilon_j)$ over all possible $\varepsilon_j$ to guarantee the unconstrained status of the firm for all possible future. Notice that the accumulation of remaining tax benefit, $T'$ enters this recursive definition, and thus firm's binary investment choice will affect the threshold that distinguishes constrained and unconstrained firms. The current dividend $D^w$ that unconstrained firms pay is

$$
\begin{aligned}
D^w(k,b,T,\varepsilon) = (1-\tau^c)\pi(k,\varepsilon) + \tau^c\delta^T T \\
&- (1-\tau^c\tau^I \mathcal{J}(K^w(k,T,\varepsilon)-(1-\delta)k))(K^w(k,T,\varepsilon)-(1-\delta)k) \\
&- (1-\tau^c\tau^b)b + q\min\{B^w(k,T,\varepsilon), \theta K^w(k,T,\varepsilon)\}
\end{aligned}
$$

Constrained firms, on the other hand, are paying negative dividends $D^w(\cdot)$ if they are adopting both unconstrained capital and bond decision rules. Therefore, as the analysis in the stylized model, I follow Jo and Senga (2019) and separate firms into *Type-1* and *Type-2* firms. Let $J^1$ denotes the value for type-1 firms and $J^2$ for type-2 firms. Type-1 firms can undertake the unconstrained level of investment but not the bond decision rules. That is to say, their bond decision are pinned down by zero dividend condition given their binary investment choice,

$$
\begin{aligned}
B^1(k,b,T,\varepsilon) = \frac{1}{q}\Big( &-(1-\tau^c)\pi(k,\varepsilon) + (1-\tau^c\tau^b)b - \tau^c\delta^T T \\
&+ (1-\tau^c\tau^I \mathcal{J}(K^w(k,T,\varepsilon)-(1-\delta)k))(K^w(k,T,\varepsilon)-(1-\delta)k)\Big),
\end{aligned}
$$

where $\mathcal{J}(I) = 1$ if $I \leq \bar{I}$, and $\xi$ otherwise. Type-1 firms are identified by $B^1(k,b,T,\varepsilon) \leq \theta K^w(k,T,\varepsilon)$. Given the $K^w(k,T,\varepsilon)$ and $B^1(k,b,T,\varepsilon)$, type-1 firms are making binary decisions between $i$-type and $a$-type,

$$
J^1(k,b,T,\varepsilon;\mu) = \max\left\{J_i^1(k,b,T,\varepsilon;\mu), J_a^1(k,b,T,\varepsilon;\mu)\right\},
$$

where $J_i^1$ and $J_a^1$ are the value function defined as

$$
\begin{aligned}
J_i^1(k,b,T,\varepsilon;\mu) &= \beta\sum_{j=1}^{N_\varepsilon}\pi_{ij}^\varepsilon V^0(k_i^*(k,T,\varepsilon), b_i^1(k_i^*), T', \varepsilon_j;\mu'), \\
b_i^1(k_i^*) &= \frac{1}{q}\Big(-(1-\tau^c)\pi(k,\varepsilon) + (1-\tau^c\tau^b)b - \tau^c\delta^T T + (1-\tau^c\tau^I\xi(k_i^*-(1-\delta)k)\Big), \\
T' &= (1-\delta^T)T + (1-\xi)(k_i^*-(1-\delta)k),
\end{aligned}
$$

and

$$
\begin{aligned}
J_a^1(k,b,T,\varepsilon;\mu) &= \beta \sum_{j=1}^{N_\varepsilon} \pi_{ij}^{\varepsilon} V^0(k_a^*(k,T,\varepsilon), b_a^1(k_a^*), T', \varepsilon_j; \mu'),\\
b_a^1(k_a^*) &= \frac{1}{q}\Big( -(1-\tau^c)\pi(k,\varepsilon) + (1-\tau^c\tau^b)b - \tau^c\delta^T T + (1-\tau^c\tau^I(k_i^* - (1-\delta)k)\Big),\\
T' &= (1-\delta^T)T.
\end{aligned}
$$

Type-2 firms, on the other hand, can follow neither unconstrained capital nor bond decision rules. They are identified by $B^1(k,b,T,\varepsilon) \geq \theta K^w(k,T,\varepsilon)$. Their bond decision is implied by binding collateral constraints, i.e., $B^2(k,b,T,\varepsilon) = \theta K^2(k,b,T,\varepsilon)$, and the capital decision $K^2(k,b,T,\varepsilon)$ has to be determined recursively. They make similar binary choice as before,

$$
J^2(k,b,T,\varepsilon) = \max\left\{ J_i^2(k,b,T,\varepsilon), J_a^2(k,b,T,\varepsilon)\right\},
$$

and $J_i^2$ and $J_a^2$ are defined as

$$
J_i^2(k,b,T,\varepsilon;\mu) = \max_{k'\in\Omega_i(k,b,T,\varepsilon)} \beta \sum_{j=1}^{N_\varepsilon} \pi_{ij}^{\varepsilon} V^0(k', b_i^2(k'), T', \varepsilon_j; \mu'),
$$

subject to

$$
\begin{aligned}
b_i^2(k') &= \frac{1}{q}\Big( -(1-\tau^c)\pi(k,\varepsilon) + (1-\tau^c\tau^b)b - \tau^c\delta^T T + (1-\tau^c\tau^I\xi(k' - (1-\delta)k)\Big),\\
T' &= (1-\delta^T)T + (1-\xi)(k'-(1-\delta)k),
\end{aligned}
$$

and

$$
J_a^2(k,b,T,\varepsilon;\mu) = \max_{k'\in\Omega_i(k,b,T,\varepsilon)} \beta \sum_{j=1}^{N_\varepsilon} \pi_{ij}^{\varepsilon} V^0(k', b_a^2(k'), T', \varepsilon_j; \mu'),
$$

subject to

$$
\begin{aligned}
b_a^2(k') &= \frac{1}{q}\Big( -(1-\tau^c)\pi(k,\varepsilon) + (1-\tau^c\tau^b)b - \tau^c\delta^T T + (1-\tau^c\tau^I(k' - (1-\delta)k)\Big),\\
T' &= (1-\delta^T)T.
\end{aligned}
$$

The choice sets for $i$-type and $a$-type firms' probelm are defined by

$$
\begin{aligned}
\Omega_i(k,b,T,\varepsilon) &= \left[(1-\delta)k + \bar{I}, \bar{k}_i(k,b,T,\varepsilon)\right],\\
\Omega_a(k,b,T,\varepsilon) &= \left[0, \max\left\{0, \min\left\{(1-\delta)k + \bar{I}, \bar{k}_a(k,b,T,\varepsilon)\right\}\right\}\right],
\end{aligned}
$$

where $\bar{k}_i$ and $\bar{k}_a$ are the maximum affordable capital given binding collateral constraints and zero dividend conditions,

$$\bar{k}_i = \frac{(1-\tau^c)\pi(k,\varepsilon) + \tau^c\delta^T T - (1-\tau^c\tau^b)b}{1-\tau^c\tau^I\xi - q\theta},$$

$$\bar{k}_a = \frac{(1-\tau^c)\pi(k,\varepsilon) + \tau^c\delta^T T - (1-\tau^c\tau^b)b}{1-\tau^c\tau^I - q\theta}.$$

# 6 Calibration and Result

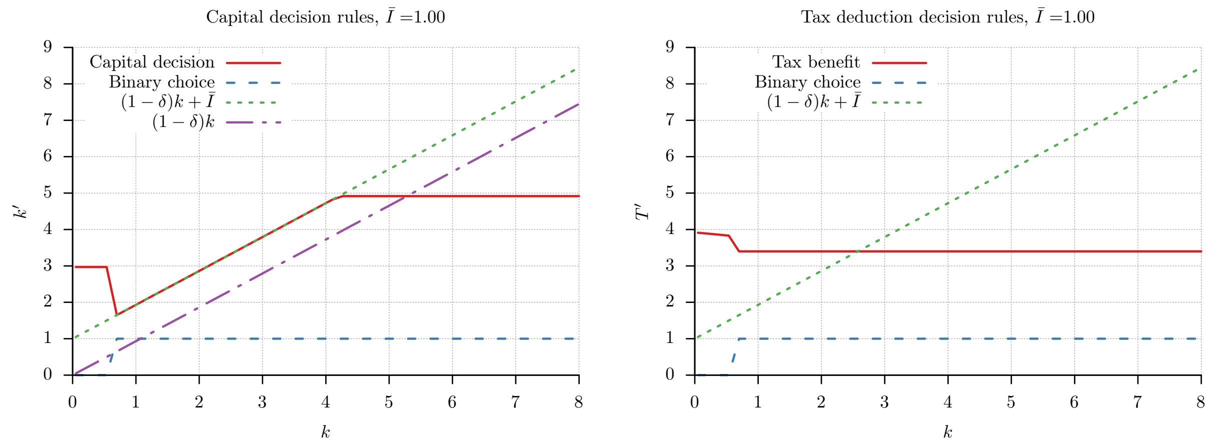
Table 2 lists the parameters I used to solve for unconstrained firms' problem. Since the distortion on capital investment by tax incentive is the core mechanics of this model, I set the length of a period to one year to match the establishment-level investment data. The functional form of the representative household's utility is assumed to be $u(c, l) = \log c + \psi l$, following Rogerson (1988). I assume Cobb-Douglas production function, $z\varepsilon F(k, n) = z\varepsilon k^{\alpha} n^{\nu}$. The initial capital $k_0$ are defined as a fraction of steady-state aggregate capital, as specified in (18), and initial bond level $b_0 = 0$. The household's discount rate $\beta$ is set to imply 4 percent of the annual interest rate. The disutility from working, $\psi$, is determined to reproduce hours of work equal to one-third. The rate of capital depreciation, $\delta$, corresponds to an investment-capital ratio of approximately 10 percent. The labor share $\nu$ is 60 percent, as shown in US postwar data.

The aggregate productivity shock $z = 1$ in the steady state. I assume the idiosyncratic productivity shock $\varepsilon$ follows log AR(1) process with persistence $\rho_{\varepsilon}$ and standard deviation $\sigma_{\eta_{\varepsilon}}$. The evolution of $\varepsilon$ is $\log \varepsilon' = \rho_{\varepsilon} \log \varepsilon + \eta'_{\varepsilon}$, with $\eta'_{\varepsilon} \sim N(0, \sigma^2_{\eta_{\varepsilon}})$. The value of $\rho_{\varepsilon}$ and $\sigma_{\eta_{\varepsilon}}$ are choosen to match the investment distribution in Cooper and Haltiwanger (2006). I will revisit these values after I solve the stationary equilibrium. Given the value specified in table 2, I use Rouwenhorst (1995) method to discretize the firm's log-normal idiosyncratic productivity process with 7 values ($N_{\varepsilon} = 7$) to obtain $\{\varepsilon_i\}_{i=1}^{N_{\varepsilon}}$ and $(\pi^{\varepsilon}_{ij})_{i,j=1}^{N_{\varepsilon}}$.

The results from the unconstrained firms' problem have similar features as in the stylized model. Figure 6 and 6 show the decision rules and comparative statics given policy tools given the current tax benefit $T$ is at the median level. The decision rules of unconstrained firms disregarding the financial frictions are presented in the left panel. The v-shaped decision rules denote the transition from $i$-type investment to $a$-type. This shows the distortion caused by maximum allowance as it shifts the inaction region from 0 investment to $\bar{I}$ investment. The difference between two steps, represented by $k_i^*$ and $k_a^*$, represents the real friction created by bonus depreciation rate $\xi$. The tax benefit decision rules are displayed in the right panel. The flat part of the tax benefit decision rules corresponds to the law of motion of tax benefit for $a$-type firms, i.e., equation (15). The future tax benefit $T'$ is determined solely by $T$ as I fixed $T$ as the median tax benefit. The kink in the right panel happens at the same capital level as the transition from $i$-type to $a$-type firms. For $a$-type firms, their tax benefit law of motion follows (9). Therefore, as current capital stock $k$ increases, firms' level of investment shrinks, and thus the downward-sloping future tax benefit $T'$ with respect to $k$.

The comparative statics of unconstrained firms' capital policy functions in figure 6 reveal the distortion that is not shown in the stylized model. In the left panel, as $\bar{I}$ increases from 0.0

Figure 3: Unconstrained firms' capital and tax benefit decision rules




to 0.1, the $a$-type target capital $k_a^*$ drop from 5.08 to 4.91. This comes from the redistribution nature of increasing the maximum allowance. Higher maximum allowance induces firms to invest lower than $\bar{I}$. When firms invest less, there is less future tax benefit $T'$ accumulated as shown in figure 6. Lower $T'$ is going to reinforce the shrink of capital through lower value function $W(\cdot)$. As a result, both the target capital $k_a^*$ and future tax benefit $T'$ drop with merely 0.1 increases in maximum allowance. Furthermore, such recursive impact is larger for $i$-type firms. An 0.1 increase of $\bar{I}$ leads to 0.16 decrease in $i$-type target capital $k_i^*$. Furthermore, as the $T'$ for $i$-type firms is increasing in investment, a drop in investment will start the vicious cycle and further decrease the target capital even when maximum allowance $\bar{I}$ increases from 0.1 to 1.0. Therefore, such recursive reinforcement from tax benefits will alter constrained firms' capital investment even more. On the contrary, the relaxation of bonus depreciation rate $\xi$ generates net gain to constrained firms. As shown in the right panel of figure 6, $i$-type firms' target capital $k_i^*$ increases when $\xi$ is higher, while the $a$-type firms' target capital $k_a^*$ remains the same. Since the bonus rate $\xi$ represents the real friction that separates $i$-type and $a$-type, a higher $\xi$ means a smaller disparity between both types of firms. Such increment in capital is supported by the accumulation of future tax benefit, as shown in figure 6.

Table 2: Parameters for quantitative model

| | Parameter | Value | Reason |
|---|---|---|---|
| *Preferences and technology* | | | |
| Discount rate | $\beta$ | 0.96 | 4% real interest rate |
| Capital share | $\alpha$ | 0.6 | private capital-output ratio |
| Labor share | $\nu$ | 0.6 | private capital-output ratio |
| Preference for leisure | $\psi$ | 2.15 | one-thrid of time endowment |
| Capital depreciation rate | $\delta$ | 0.069 | average investment-capital ratio |
| *Shocks and firm characteristics* | | | |
| Persistence of $\varepsilon$ | $\rho_\varepsilon$ | 0.7 | investment distribution moments |
| Standard deviation of $\varepsilon$ | $\sigma_{\eta_\varepsilon}$ | 0.12 | investment distribution moments |
| exogenous exit rate | $\pi_d$ | 0.1 | 10% entry and exit |
| fraction of entrants capital endowment | $\chi$ | 0.1 | 10% of aggregate capital |
| Collateralizability | $\theta$ | 0.5 | Li, Whited and Wu (2012) |
| *Policy tools* | | | |
| Corporate tax rate | $\tau^c$ | 0.21 | US Tax schedule after TCJA |
| Investment tax benefit | $\tau^I$ | 0.35 | |
| Tax benefit depreciation rate | $\delta^T$ | 0.138 | $\delta^T = 2\delta$ |
| Bonus depreciation rate in baseline | $\xi$ | 0.7 | |
| Maximum allowance | $\bar{I}$ | 1.0 | |

Figure 4: Comparative statics of unconstrained firms' capital decision rules

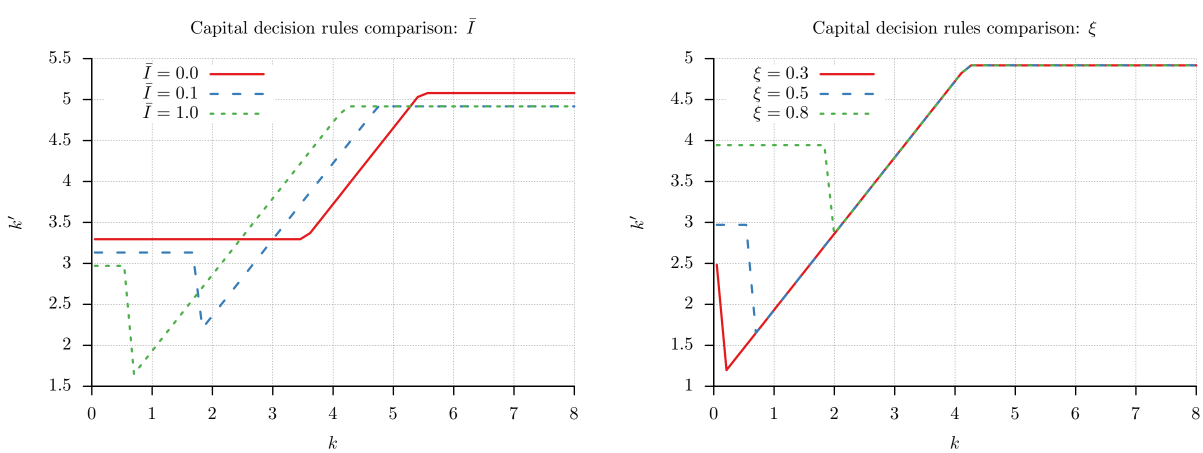
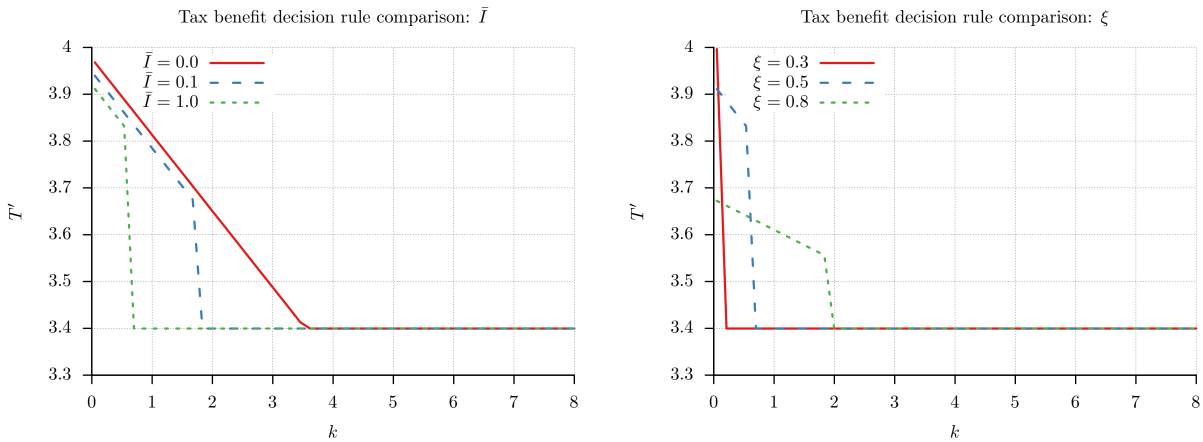


Figure 5: Comparative statics for unconstrained firms' tax benefit decision rules




# 7 Concluding Remarks

I have developed a general equilibrium model with heterogeneous firms and collateral constraints to evaluate the efficacy of investment subsidy policies. Contrary to ordinary policies, both investment subsidy policies, the maximum allowance and bonus depreciation, are utilized in the form of corporate tax deductions in the United States. By properly addressing this characteristic, the stylized model qualitatively replicates the cross-sectional features of investment response documented in the empirical literature. I showed that both investment subsidy policies have significantly different implications for the firm investment. Increasing the rate of accelerated depreciation to the bonus rate, on the one hand, generates the net gain to the firms. This policy allows firms who undertake large investments to enjoy the tax deduction today without distorting the incentive for those who invest in small amounts. On the other hand, increasing the maximum allowance distorts investment incentives. Firms that originally aim for higher target capital only invest up to the maximum allowance to be qualified for full tax deduction. Such distortion is amplified by the financial constraints, and the constrained firms are willing to sacrifice capital accumulation and relaxation of collateral constraints in exchange for immediate tax deductions. As a result, maximum allowance backfires its policy goal and hinders the small firms rather than helps them.

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